Experimental Design and Statistical Inference for Conjoint Analysis: The Essential Role of Population Distribution

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Conjoint Analysis and Average Marginal Component Effect

- Conjoint analysis: experiments to study multidimensional preferences
- Evaluate two profiles that have randomized characteristics
- Example: political candidates with varying gender, party, policy, ...

- Main causal estimand: Average Marginal Component Effect (AMCE)
  - causal effect of a factor while marginalizing the other factors
  - effect of being female while marginalizing over party, policy...

- AMCE = a weighted average of causal effects across different profiles
  (not just average across respondents)

- **Problem**: Choice of marginalizing distribution has been ignored
- More than 90% of the papers use uniform randomization
  - ⇝ equal weights given to all possible profiles
  - ⇝ undermine the external validity of the AMCE

Population AMCE (pAMCE)

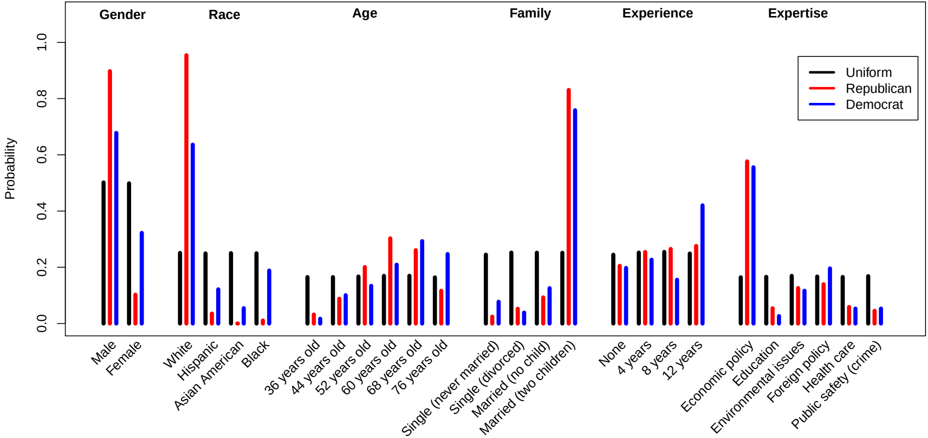
- Use population distribution when marginalizing over the other factors
- Distribution of actual candidates' attributes (party, policy,...)
- Effects in the real world $\rightsquigarrow$ higher external validity

Three Contributions:

1. Compare the pAMCE and the AMCE based on uniform distribution
2. Design-based Confirmatory Analysis
   - Incorporate population distribution at the design stage
   - New experimental design and estimator
3. Model-based Exploratory Analysis
   - Explore the pAMCEs using existing conjoint data
   - Flexible two-way interaction model

The Effect of Gender on Voter Choice: Ono & Burden (2018)

- voters choose hypothetical candidates with varying 13 factors
- Original: all factors are randomized with uniform
- Population distribution based on Republican & Democratic legislators



Potential Outcomes for Conjoint Analysis

- Respondent $i \in \{1, \ldots, N\}$ evaluates K choice tasks ($K = 10$)
- Within each task, pick one of J profiles ($J = 2$)
- Profile is characterized by L factors ($L = 13$) (e.g., gender and race)
- Within factor ℓ, the number of levels is D_ℓ (e.g., $D_\ell = 2$ for gender)

- Profile : vector $\mathbf{T}_{ijk}$ of length L
- ℓth factor of profile: $\mathbf{T}_{ijk\ell}$ (e.g., Gender = female)
- Potential outcomes: $Y_{ijk}(\mathbf{t}_{ijk}, \mathbf{t}_{i,-j,k})$ (e.g, choose jth candidate)

  $\mathbf{t}_{ijk}$: characteristics of candidate j

  $\mathbf{t}_{i,-j,k}$: characteristics of the other candidates

- Under No Profile-Order Effects, $Y_{ijk}(\mathbf{t}_{ijk}, \mathbf{t}_{i,-j,k}) = Y_{ik}(\mathbf{t}_{ijk}, \mathbf{t}_{i,-j,k})$

Average Marginal Component Effect (AMCE; HHY 2014)

- AMCE: the average causal effect of changing levels within factor ℓ while marginalizing the other factors

$$\tau_\ell(t_1, t_0; \Pr(\mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k}))$$

$$= \sum_{\mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k}} \underbrace{\mathbb{E}\left[Y_{ik}(t_1, \mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k}) - Y_{ik}(t_0, \mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})\right]}_{\text{Causal Effect of factor } \ell \text{ for profile } (\mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})} \underbrace{\Pr(\mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})}_{\text{Marginalizing distribution}}$$

- Effect of being female relative to male while marginalizing Party and Abortion stance
- AMCE = an average of
  - DC: Effect of gender for candidate {Democrat, pro-choice}
  - DL: Effect of gender for candidate {Democrat, pro-life}
  - RC: Effect of gender for candidate {Republican, pro-choice}
  - RL: Effect of gender for candidate {Republican, pro-life}
- weights for each effect ← marginalizing distribution

Uniform AMCE and Population AMCE

- Uniform randomization + Difference-in-means $\rightsquigarrow$ uniform AMCE

$$\text{uAMCE} = \tau_\ell(t_1, t_0; \Pr^{\mathrm{U}}(\mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})) \quad (1)$$

- the uAMCE $= 0.25\texttt{DC} + 0.25\texttt{DL} + 0.25\texttt{RC} + 0.25\texttt{RL}$
- **Problem**: equal weights to all profiles regardless of realism
  $\rightsquigarrow$ External validity of the uAMCE is low

- New causal quantity of interest: population AMCE

$$\text{pAMCE} = \tau_\ell(t_1, t_0; \underbrace{\Pr^{*}(\mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})}_{\text{Target population distribution}}) \quad (2)$$

- the pAMCE $= 0.45\texttt{DC} + 0.01\texttt{DL} + 0.01\texttt{RC} + 0.53\texttt{RL}$

- Bias of the uAMCE relative to the pAMCE
  (1) the difference between uniform and population distributions
  (2) causal interaction between the primary factor and the other factors

Design-based Confirmatory Analysis

- Goal: test several pre-specified hypotheses about the pAMCEs
- Idea: incorporate target population distribution at the design stage

- Main factors $\mathbf{T}^{\mathcal{M}}$ (e.g., gender and race) $\leftarrow$ estimate the pAMCEs
- Control factors $\mathbf{T}^{\mathcal{C}}$ (e.g., age and party) $\leftarrow$ control variables

- **Mixed Randomization Design:**
  (1) Randomize main factors $\mathbf{T}^{\mathcal{M}}$ with uniform distributions
  (2) Randomize control factors $\mathbf{T}^{\mathcal{C}}$ with their population distributions

- More main factors $\rightarrow$ standard error for each pAMCE is larger

Weighted Difference-in-Means

- Weighted difference-in-means estimator

$$\frac{\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K}\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}Y_{ijk}}{\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K}\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}} - \frac{\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K}\mathbf{1}\{T_{ijk\ell}=t_0\}w_{ijk\ell}Y_{ijk}}{\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K}\mathbf{1}\{T_{ijk\ell}=t_0\}w_{ijk\ell}},$$

where the weights are

$$w_{ijk\ell} = \underbrace{\frac{1}{\Pr^{\mathcal{R}}(T_{ijk\ell} \mid \mathbf{T}_{ijk,-\ell}, \mathbf{T}_{i,-j,k})}}_{\text{Inverse probability of receiving } T_{ijk\ell}} \times \underbrace{\frac{\Pr^{*}(\mathbf{T}_{ijk,-\ell}, \mathbf{T}_{i,-j,k})}{\Pr^{\mathcal{R}}(\mathbf{T}_{ijk,-\ell}, \mathbf{T}_{i,-j,k})}}_{\substack{\text{Differences in distributions} \\ \text{of the other factors}}}$$

- Mixed design $\rightarrow w_{ijk\ell} = 1$ when the only main factor is ℓ
- Mixed randomization + Difference-in-means $\rightsquigarrow$ the population AMCE
- Implement via a weighted linear regression

- Population distributions can be approximated by separate marginal distributions, e.g., $\Pr^*(\text{Gender}, \text{Age}) \approx \Pr^*(\text{Gender}) \Pr^*(\text{Age})$
- No approximation error unless strong three-way interaction effects

Model-based Exploratory Analysis

- Goal: explore the pAMCEs using previous conjoint experiments
- Idea: use models to adjust for differences in uniform and population

- Latent utility model (paired-choice):

$$\widetilde{Y}_{ijk}(\mathbf{T}_{ijk}, \mathbf{T}_{ij'k}) = \widetilde{\alpha} + \underbrace{\sum_{\ell=1}^{L} \mathbf{X}_{ijk\ell}^{\top} \widetilde{\beta}_{\ell} + \sum_{\ell,\ell'} (\mathbf{X}_{ijk\ell} \bigtimes \mathbf{X}_{ijk\ell'})^{\top} \widetilde{\gamma}_{\ell\ell'}}_{\text{Effect of factors of own profile}}$$

$$\underbrace{-\sum_{\ell=1}^{L} \mathbf{X}_{ij'k\ell}^{\top} \widetilde{\beta}_{\ell} - \sum_{\ell,\ell'} (\mathbf{X}_{ij'k\ell} \bigtimes \mathbf{X}_{ij'k\ell'})^{\top} \widetilde{\gamma}_{\ell\ell'}}_{\text{Effect of factors of the opponent}} + \underbrace{\sum_{\ell=1}^{L} (\mathbf{X}_{ijk\ell} \bigtimes \mathbf{X}_{ij'k\ell})^{\top} \widetilde{\delta}_{\ell\ell}}_{\text{Interaction across profiles}} + \widetilde{\epsilon}_{ijk},$$

- Linear probability model (Egami and Imai, 2019)

$$\Pr(Y_{ik} = 1 \mid \mathbf{T}_{ijk}, \mathbf{T}_{ij'k}) = \alpha + \sum_{\ell} (\mathbf{X}_{ijk\ell} - \mathbf{X}_{ij'k\ell})^{\top} \underbrace{\beta_{\ell}}_{\text{Main effects}}$$

$$+ \sum_{\ell,\ell'} (\mathbf{X}_{ijk\ell} \bigtimes \mathbf{X}_{ijk\ell'} - \mathbf{X}_{ij'k\ell} \bigtimes \mathbf{X}_{ij'k\ell'})^{\top} \underbrace{\gamma_{\ell\ell'}}_{\text{within-profile Interaction}} + \sum (\mathbf{X}_{ijk\ell} \bigtimes \mathbf{X}_{ij'k\ell})^{\top} \underbrace{\delta_{\ell\ell}}_{\text{cross-profile Interaction}}$$

Estimating the Population AMCE

- Estimating the pAMCE

$$\widehat{\tau}^*_\ell(t_1, t_0) = \underbrace{\widehat{\beta}_{\ell 1}}_{\text{Main effect}} + \sum_{\ell'=1}^{L} \sum_{d=1}^{D_{\ell'}-1} \underbrace{\widehat{\gamma}_{\ell 1 \ell' d}}_{\substack{\text{Interaction} \\ \text{effects}}} \times \underbrace{\Pr^*(T_{ijk\ell'} = d)}_{\substack{\text{Marginal distribution} \\ \text{of population}}}$$

- Bias of the uAMCE relative to the pAMCE (decomposition)

$$\widehat{\text{Bias}} = \sum_{\ell'=1}^{L} \widehat{\text{Bias}}_{\ell'} = \sum_{\ell'=1}^{L} \sum_{d=1}^{D_{\ell'}-1} \underbrace{\widehat{\gamma}_{\ell 1 \ell' d}}_{\substack{\text{Interaction} \\ \text{effects}}} \underbrace{\{\Pr^*(T_{ijk\ell'} = d) - \Pr^{\mathrm{U}}(T_{ijk\ell'} = d)\}}_{\text{Difference in distributions}}$$

- Standard errors are large $\leftarrow$ Need to estimate many interactions
- Regularization to collapse redundant levels $\leftarrow$ Improve efficiency

The Effect of Gender on Voter Choice: Ono & Burden (2018)

- Voters choose one of the two hypothetical candidates
- Candidates differ in 13 factors

  `Gender, Race, Age, Family, Experience, Expertise, Trait, Party, Office, Immigration Policy, Security Policy, Abortion Policy, Deficit Policy, Favorability Rating`
- Each factor is randomized with uniform distribution
- Interested in the AMCE of being female relative to male

Two Main Findings from Original Analysis:

(1) Find the uniform AMCE to be a small negative effect

$\rightarrow$ Bias against female candidates exists but small

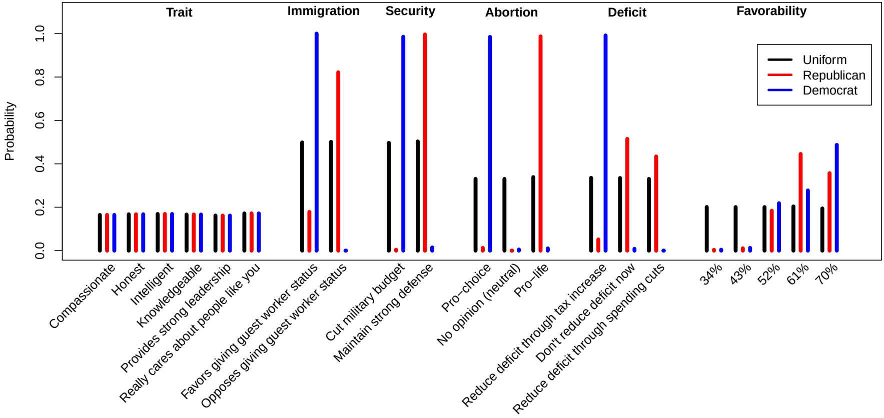
(2) The uniform AMCE is negative for presidential candidates but no effect for congressional candidates

$\rightarrow$ Bias against female candidates only for presidential candidates

- Revisit the findings by estimating the population AMCEs

Building Population Distributions

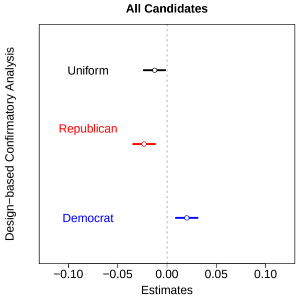
- Target Population: 115th U.S. Congress
- Data sources: (1) biographical dataset (2) electoral returns (3) issue-based scorecards (4) actual bills (5) Wikipedia (6) TheHill.com
- Population distributions separately for Democrats and Republican



Design-based Confirmatory Analysis

- Goal: test pre-specified hypotheses about the effect of gender
- Mixed randomization design
  - ⋆ Uniform distribution for `Gender` — factor of interest
  - ⋆ Population distribution for the remaining 12 factors
- Original experiments are done with Uniform
- → Illustrate with simulations based on the original conjoint data
  - (1) Fit the two-way interaction model
  - (2) Take estimated coefficients as true data generating process
  - (3) Randomize profiles according to mixed design
  - (4) Estimate pAMCEs with weighted diff-in-means estimator
- Three Estimands: all profiles, congressional, presidential
- If mixed design had been used, how would conclusions change?

Real World Distribution Leads to Different Findings



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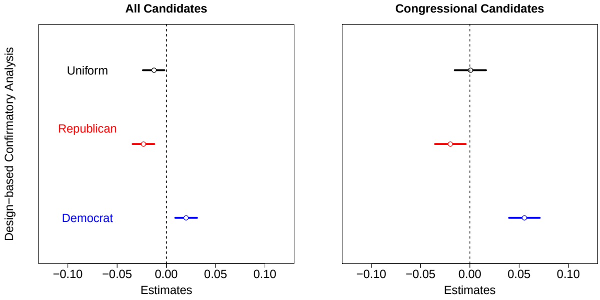

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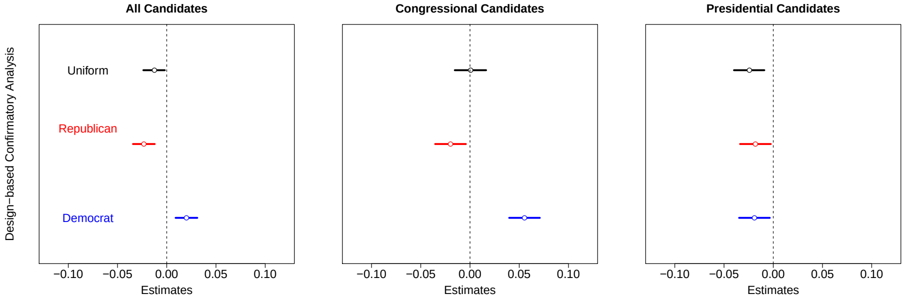
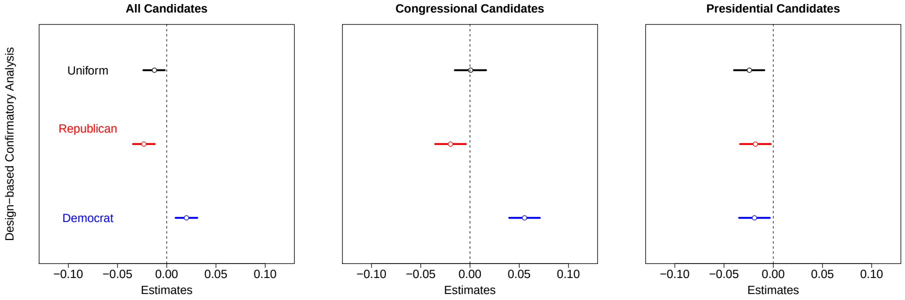


Model-based Exploratory Analysis

- Goal: explore pAMCEs using existing conjoint experiments
- Fit linear probability model with all the two-way interactions
- Regularization to collapse levels $\rightarrow$ improve efficiency
- Expect that standard errors are larger
- Unpacking the bias of the uAMCE relative to the pAMCE
  (1) Decompose bias into each factor
  (2) Is bias caused by interactions or difference in distributions?

Real World Distribution Leads to Different Findings



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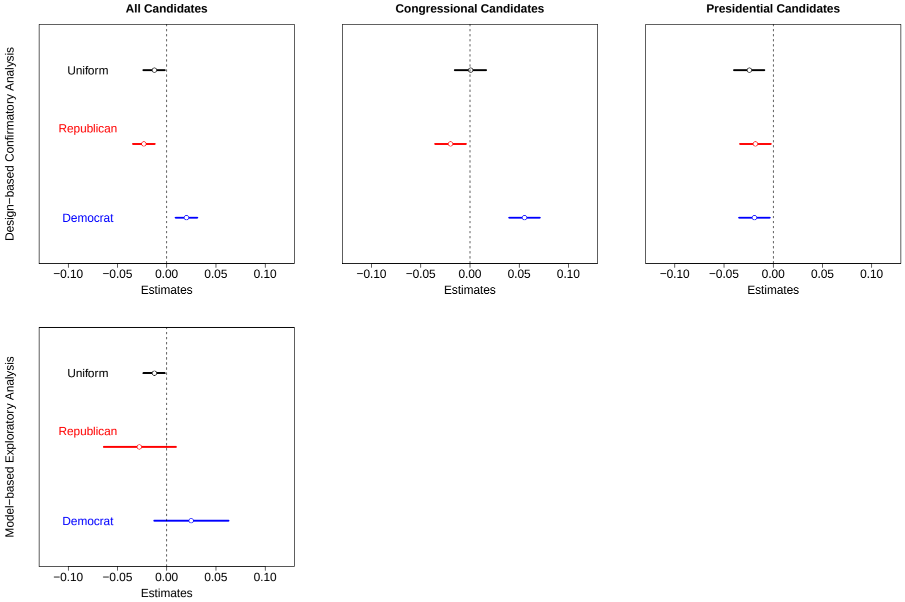

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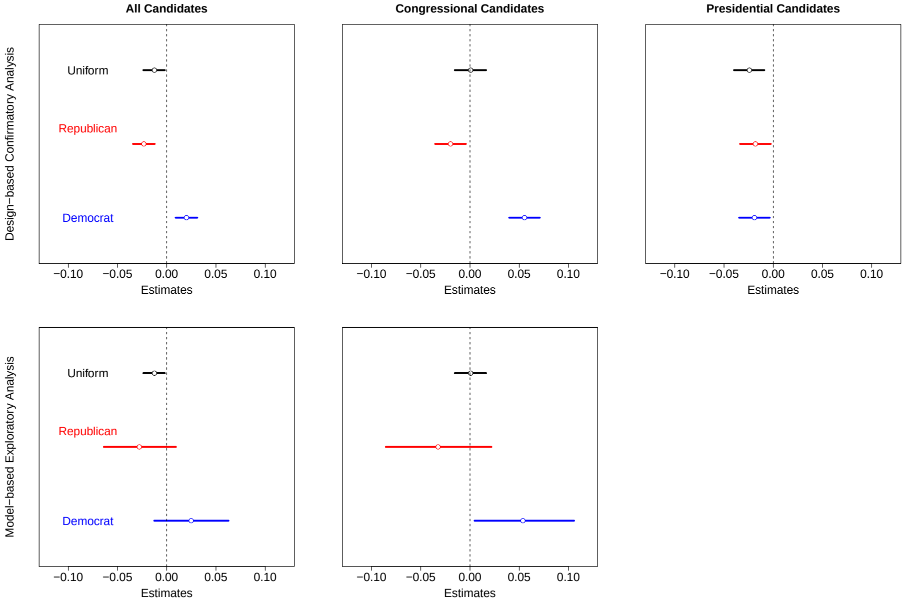

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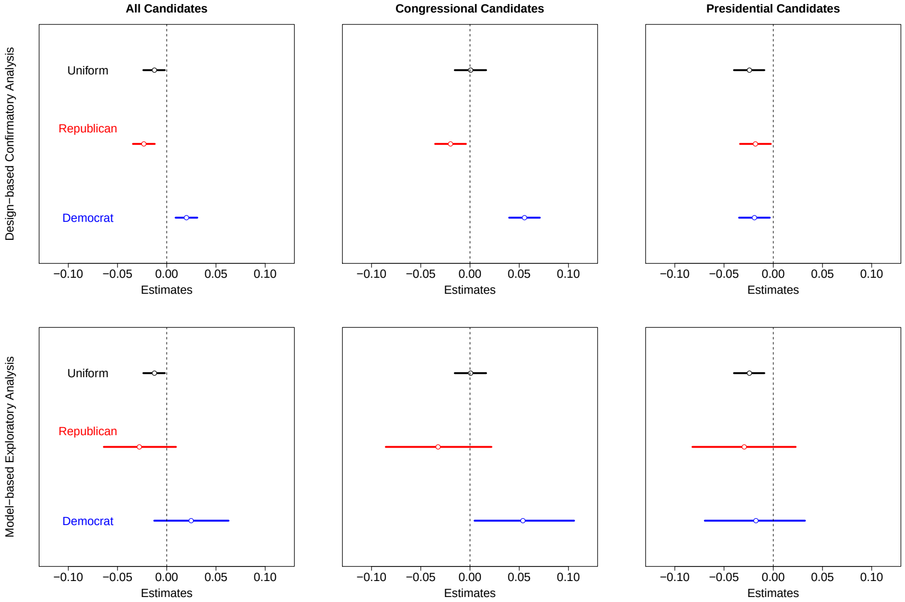
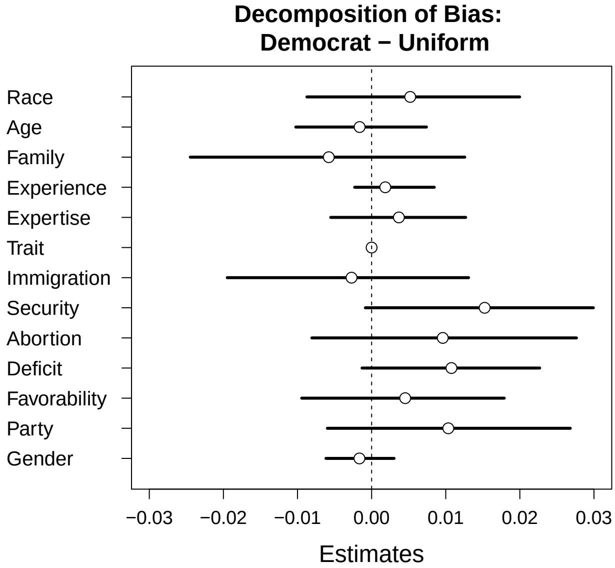
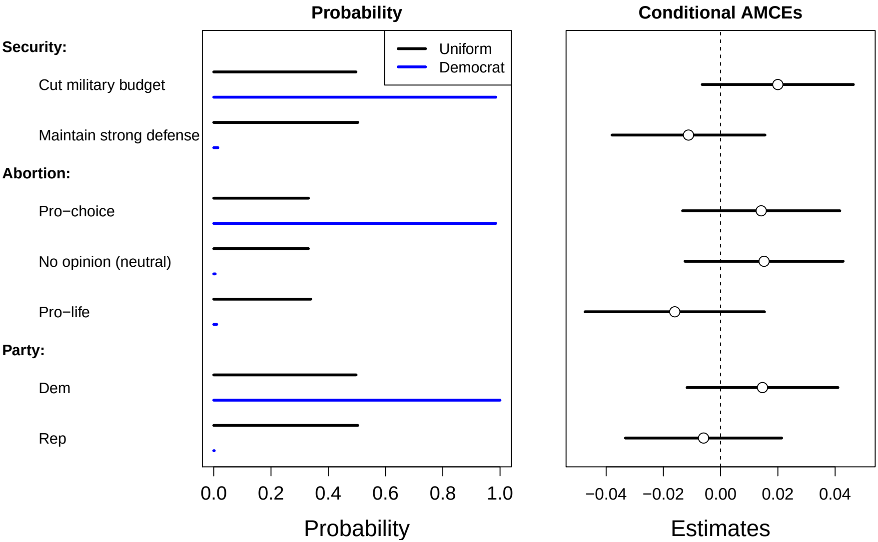


Bias Decomposition for Democratic Congressional pAMCE



Bias in Uniform AMCE Caused By Distribution



Concluding Remarks

- AMCE critically depends on the marginalizing distribution
- **Problem**: Uniform distribution is used for convenience
- uAMCE has low external validity $\leftarrow$ equal weights to all profiles

- Population AMCE: marginalize over population distribution

  (1) the real-world distributions of actual politicians' attributes

  (2) counterfactual distributions of information environments

- Two methods to estimate Population AMCE

  Developing an open-source software `R` package

- External validity of treatments (profiles) are essential

  Conjoint mimics real-world *outcome*

  Population distribution essential to treatment realism

References

- de la Cuesta, Egami, and Imai. (2019). "Experimental Design and Statistical Inference for Conjoint Analysis: The Essential Role of Population Distribution."
- Egami and Imai. (2019). "Causal Interaction in Factorial Experiments: Application to Conjoint Analysis," *Journal of the American Statistical Association*.

Send comments and suggestions to us

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The Role of Information Environment Peterson (2017)

- Voters choose between one of two candidates
- Candidates differ along 10 dimensions:

  `Age, Gender, Race, Education, Profession, Family, Military Service, Party, Abortion Stance, Spending Stance`
- Three-step randomization:

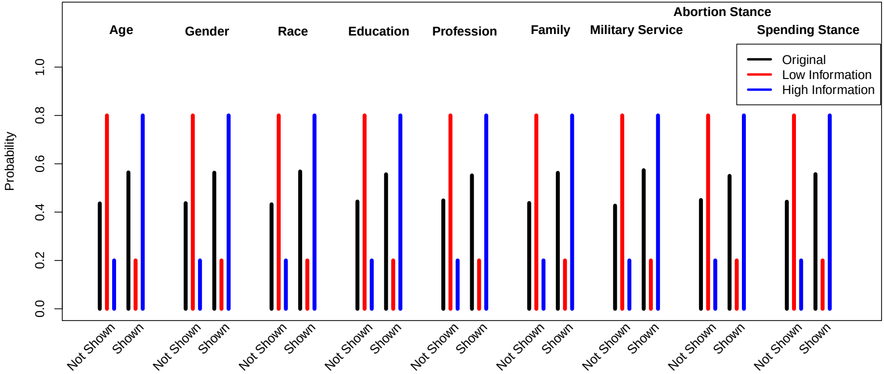
  Number of factors, which factors, which level

  Main finding from original analysis:

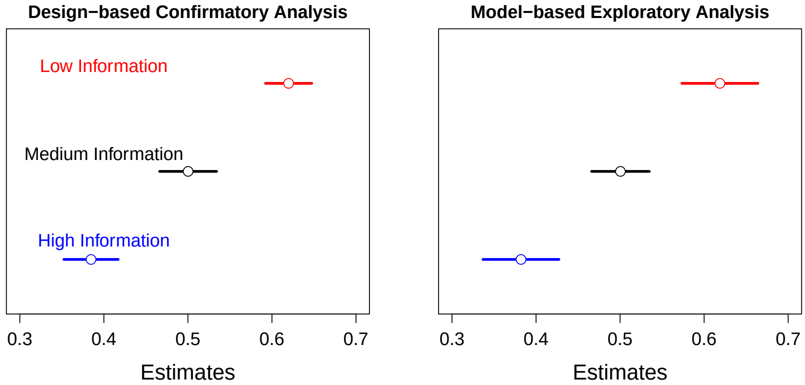
  (1) Effect of copartisanship depends on environment

  (2) Less information $\rightarrow$ copartisanship matters more

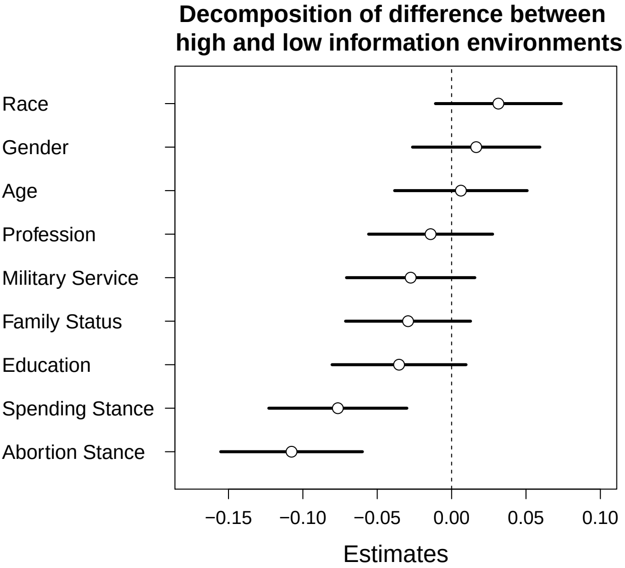
A Counterfactual Information Environment



Information Environment Matters



Bias Decomposition



Effect Comes from Large Interactions