When Should We Use Linear Fixed Effects Regression Models for Causal Inference with Panel Data?

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Fixed Effects Regressions in Causal Inference

- Linear fixed effects regression models are the primary workhorse for causal inference with panel data
- Researchers use them to adjust for unobserved confounders (omitted variables, endogeneity, selection bias, ...):
  - "Good instruments are hard to find ..., so we'd like to have other tools to deal with unobserved confounders. This chapter considers ... strategies that use data with a time or cohort dimension to control for unobserved but fixed omitted variables" (Angrist & Pischke, *Mostly Harmless Econometrics*)
  - "fixed effects regression can scarcely be faulted for being the bearer of bad tidings" (Green *et al.*, *Dirty Pool*)

Motivating Questions

1. What make it possible for fixed effects regression models to adjust for unobserved confounding?
2. Are there any trade-offs when compared to the selection-on-observables approaches such as matching?
3. What are the exact causal assumptions underlying fixed effects regression models?

Main Results of the Paper

- Identify causal assumptions of one-way fixed effects estimators:
  1. Treatments do not directly affect future outcomes
  2. Outcomes do not directly affect future treatments and future time-varying confounders

  $\rightsquigarrow$ can be relaxed under the selection-on-observables approach
- Develop within-unit matching estimators to relax the functional form assumptions of linear fixed effects regression estimators
- Identify the problem of two-way fixed effects regression models
  $\rightsquigarrow$ no other observations share the same unit and time
- Propose simple ways to improve fixed effects estimators using the new matching/weighted fixed effects regression framework
- Replace the assumptions with the design-based assumptions
  $\rightsquigarrow$ before-and-after and difference-in-differences designs

Linear Regression with Unit Fixed Effects

- Balanced panel data with N units and T time periods
- Y_{it}: outcome variable
- X_{it}: causal or treatment variable of interest

- Model:

$$Y_{it} = \alpha_i + \beta X_{it} + \epsilon_{it}$$

- Estimator: "de-meaning"

$$\hat{\beta}_{\text{FE}} = \operatorname*{arg\,min}_{\beta} \sum_{i=1}^{N} \sum_{t=1}^{T} \{(Y_{it} - \overline{Y}_i) - \beta(X_{it} - \overline{X}_i)\}^2$$

where $\overline{X}_i$ and $\overline{Y}_i$ are unit-specific sample means

The Standard Assumption

Assumption 1 (Strict Exogeneity)

$$\mathbb{E}(\epsilon_{it} \mid \mathbf{X}_i, \alpha_i) \quad = \quad 0$$

where $\mathbf{X}_i$ *is a* $T \times 1$ *vector of treatment variables for unit* i

- $\mathbf{U}_i$: a vector of time-invariant unobserved confounders
- $\alpha_i = h(\mathbf{U}_i)$ for *any* function $h(\cdot)$
- A flexible way to adjust for unobservables

Causal Assumption I

Assumption 2 (No carryover effect)

Treatments do not directly affect future outcomes

$$Y_{it}(X_{i1}, X_{i2}, \ldots, X_{i,t-1}, X_{it}) \quad = \quad Y_{it}(X_{it})$$

- Potential outcome model:

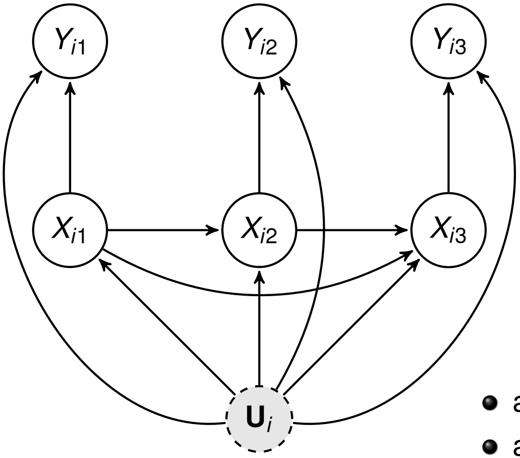
$$Y_{it}(x) \quad = \quad \alpha_i + \beta x + \epsilon_{it} \quad \text{for } x = 0, 1$$

- Average treatment effect:

$$\tau \quad = \quad \mathbb{E}(Y_{it}(1) - Y_{it}(0) \mid C_i = 1) \quad = \quad \beta$$

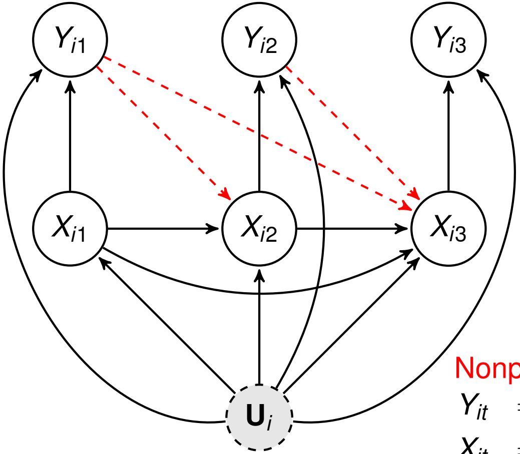
where $C_i = \mathbf{1}\{0 < \sum_{t=1}^{T} X_{it} < T\}$

Causal Directed Acyclic Graph (DAG)



- arrow $=$ direct causal effect
- absence of arrows
  $\rightsquigarrow$ causal assumptions

Causal Directed Acyclic Graph (DAG)



Adding a red dashed arrow violates strict exogeneity

Nonparametric SEM (Pearl)

$$Y_{it} = g_1(X_{it}, \mathbf{U}_i, \epsilon_{it})$$
$$X_{it} = g_2(X_{i1}, \ldots, X_{i,t-1}, \mathbf{U}_i, \eta_{it})$$

Causal Assumption II

- What randomized experiment satisfies strict exogeneity?

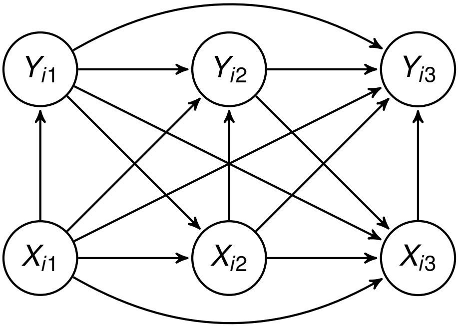
Assumption 3 (Sequential Ignorability with Unobservables)

$$
\begin{aligned}
\{Y_{it}(1), Y_{it}(0)\}_{t=1}^{T} \quad &\perp\!\!\!\perp \quad X_{i1} \mid \mathbf{U}_i \\
&\vdots \\
\{Y_{it}(1), Y_{it}(0)\}_{t=1}^{T} \quad &\perp\!\!\!\perp \quad X_{it'} \mid X_{i1}, \ldots, X_{i,t'-1}, \mathbf{U}_i \\
&\vdots \\
\{Y_{it}(1), Y_{it}(0)\}_{t=1}^{T} \quad &\perp\!\!\!\perp \quad X_{iT} \mid X_{i1}, \ldots, X_{i,T-1}, \mathbf{U}_i
\end{aligned}
$$

- The "as-if random" assumption without conditioning on the previous outcomes
- Outcomes can *directly* affect future outcomes $\rightsquigarrow$ but no need to adjust for past outcomes
- Nonparametric identification result

An Alternative Selection-on-Observables Approach

- Marginal structural models in epidemiology (Robins)
- Risk set matching (Rosenbaum)
- Trade-off: unobserved time-invariant confounders vs. direct effect of outcome on future treatment



Within-Unit Matching Estimator

- Even if these assumptions are satisfied, the the unit fixed effects estimator is inconsistent for the ATE:

$$\hat{\beta}_{\text{FE}} \quad \xrightarrow{p} \quad \frac{\mathbb{E}\left\{C_i\left(\frac{\sum_{t=1}^{T} X_{it}Y_{it}}{\sum_{t=1}^{T} X_{it}} - \frac{\sum_{t=1}^{T}(1-X_{it})Y_{it}}{\sum_{t=1}^{T} 1-X_{it}}\right)S_i^2\right\}}{\mathbb{E}(C_i S_i^2)} \quad \neq \quad \tau$$

  where $S_i^2 = \sum_{t=1}^{T}(X_{it} - \overline{X}_i)^2/(T-1)$ is the unit-specific variance

- The Within-unit matching estimator improves $\hat{\beta}_{\text{FE}}$ by relaxing the linearity assumption:

$$\hat{\tau}_{\text{match}} \quad = \quad \frac{1}{\sum_{i=1}^{N} C_i}\sum_{i=1}^{N} C_i\left(\frac{\sum_{t=1}^{T} X_{it}Y_{it}}{\sum_{t=1}^{T} X_{it}} - \frac{\sum_{t=1}^{T}(1-X_{it})Y_{it}}{\sum_{t=1}^{T}(1-X_{it})}\right)$$

Constructing a General Matching Estimator

- $\mathcal{M}_{it}$: matched set for observation (i, t)
- For the within-unit matching estimator,

$$\mathcal{M}(i,t) \quad = \quad \{(i',t') : i' = i, X_{i't'} = 1 - X_{it}\}$$

- A general matching estimator just introduced:

$$\hat{\tau}_{\text{match}} \quad = \quad \frac{1}{\sum_{i=1}^{N}\sum_{t=1}^{T} D_{it}} \sum_{i=1}^{N}\sum_{t=1}^{T} D_{it}(\widehat{Y_{it}(1)} - \widehat{Y_{it}(0)})$$

where $D_{it} = \mathbf{1}\{\#\mathcal{M}(i,t) > 0\}$ and

$$\widehat{Y_{it}(x)} \quad = \quad \begin{cases} Y_{it} & \text{if } X_{it} = x \\ \frac{1}{\#\mathcal{M}(i,t)} \sum_{(i',t') \in \mathcal{M}(i,t)} Y_{i't'} & \text{if } X_{it} = 1 - x \end{cases}$$

Unit Fixed Effects Estimator as a Matching Estimator

- "de-meaning" $\rightsquigarrow$ match with all other observations within the same unit:

$$\mathcal{M}(i,t) \;=\; \{(i',t') : i' = i, t' \neq t\}$$

- mismatch: observations with the same treatment status
- Unit fixed effects estimator adjusts for mismatches:

$$\hat{\beta}_{\text{FE}} \;=\; \frac{1}{K}\left\{\frac{1}{\sum_{i=1}^{N}\sum_{t=1}^{T} D_{it}}\sum_{i=1}^{N}\sum_{t=1}^{T} D_{it}\left(\widehat{Y_{it}(1)} - \widehat{Y_{it}(0)}\right)\right\}$$

  where K is the proportion of proper matches

- The within-unit matching estimator eliminates all mismatches

Matching as a Weighted Unit Fixed Effects Estimator

- *Any* within-unit matching estimator can be written as a weighted unit fixed effects estimator with different regression weights

- The proposed within-matching estimator:

$$\hat{\beta}_{\text{WFE}} = \underset{\beta}{\arg\min} \sum_{i=1}^{N} \sum_{t=1}^{T} D_{it} W_{it} \{(Y_{it} - \overline{Y}_i^*) - \beta(X_{it} - \overline{X}_i^*)\}^2$$

where $\overline{X}_i^*$ and $\overline{Y}_i^*$ are unit-specific weighted averages, and

$$W_{it} = \begin{cases} \frac{T}{\sum_{t'=1}^{T} X_{it'}} & \text{if} \quad X_{it} = 1, \\ \frac{T}{\sum_{t'=1}^{T} (1 - X_{it'})} & \text{if} \quad X_{it} = 0. \end{cases}$$

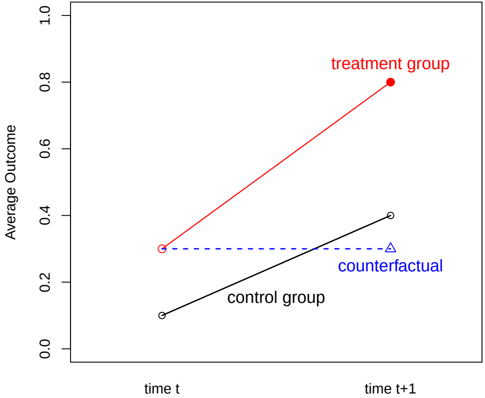
- We show how to construct regression weights for different matching estimators (i.e., different matched sets)
- Idea: count the number of times each observation is used for matching

- Benefits:
  - computational efficiency
  - model-based standard errors
  - double-robustness $\rightsquigarrow$ matching estimator is consistent even when linear fixed effects regression is the true model
  - specification test (White 1980) $\rightsquigarrow$ null hypothesis: linear fixed effects regression is the true model

Before-and-After Design

- The assumption that outcomes do not directly affect future treatments may not be credible
- Replace it with the design-based assumption:

$$\mathbb{E}(Y_{it}(x) \mid X_{it} = x') \quad = \quad \mathbb{E}(Y_{i,t-1}(x) \mid X_{i,t-1} = 1 - x')$$



- This is a matching estimator with the following matched set:

$$\mathcal{M}(i,t) = \{(i',t') : i' = i, t' \in \{t-1, t+1\}, X_{i't'} = 1 - X_{it}\}$$

- It is also the first differencing estimator:

$$\hat{\beta}_{\text{FD}} = \underset{\beta}{\arg\min} \sum_{i=1}^{N} \sum_{t=2}^{T} \{(Y_{it} - Y_{i,t-1}) - \beta(X_{it} - X_{i,t-1})\}^2$$

- "We emphasize that the model and the interpretation of β are *exactly* as in [the linear fixed effects model]. What differs is our method for estimating β" (Wooldridge; italics original).
- The identification assumptions is very different!

Remarks on Other Important Issues

1. Adjusting for observed time-varying confounding $\mathbf{Z}_{it}$
   - Proposes within-unit matching estimators that adjust for $\mathbf{Z}_{it}$
   - Key assumption: outcomes neither directly affect future treatments nor future time-varying confounders
2. Adjusting for past treatments
   - Impossible to adjust for all past treatments within the same unit
   - Researchers must decide the number of past treatments to adjust
3. Adjusting for past outcomes
   - No need to adjust for past outcomes if they do not directly affect future treatments
   - If they do, the strict exogeneity assumption will be violated
   - Past outcomes as instrumental variables (Arellano and Bond) $\rightsquigarrow$ often not credible

No free lunch: adjustment for unobservables comes with costs

Linear Regression with Unit and Time Fixed Effects

- Model:

$$Y_{it} = \alpha_i + \gamma_t + \beta X_{it} + \epsilon_{it}$$

  where γ_t flexibly adjusts for a vector of unobserved unit-invariant time effects $\mathbf{V}_t$, i.e., $\gamma_t = f(\mathbf{V}_t)$

- Estimator:

$$\hat{\beta}_{\text{FE2}} = \arg\min_{\beta} \sum_{i=1}^{N} \sum_{t=1}^{T} \{(Y_{it} - \overline{Y}_i - \overline{Y}_t + \overline{Y}) - \beta(X_{it} - \overline{X}_i - \overline{X}_t + \overline{X})\}^2$$

  where $\overline{Y}_t$ and $\overline{X}_t$ are time-specific means, and $\overline{Y}$ and $\overline{X}$ are overall means

Understanding the Two-way Fixed Effects Estimator

- β_{FE}: bias due to time effects
- β_{FEtime}: bias due to unit effects
- β_{pool}: bias due to both time and unit effects

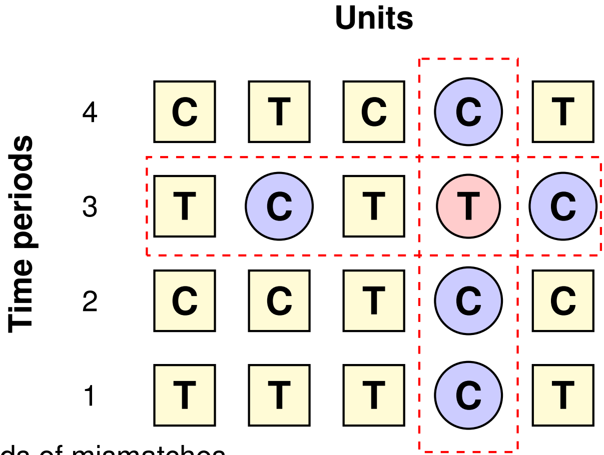
$$\hat{\beta}_{\text{FE2}} = \frac{\omega_{\text{FE}} \times \hat{\beta}_{\text{FE}} + \omega_{\text{FEtime}} \times \hat{\beta}_{\text{FEtime}} - \omega_{\text{pool}} \times \hat{\beta}_{\text{pool}}}{w_{\text{FE}} + w_{\text{FEtime}} - w_{\text{pool}}}$$

with sufficiently large N and T, the weights are given by,

$$\begin{aligned}
\omega_{\text{FE}} &\approx \mathbb{E}(S_i^2) = \text{average unit-specific variance} \\
\omega_{\text{FEtime}} &\approx \mathbb{E}(S_t^2) = \text{average time-specific variance} \\
\omega_{\text{pool}} &\approx S^2 = \text{overall variance}
\end{aligned}$$

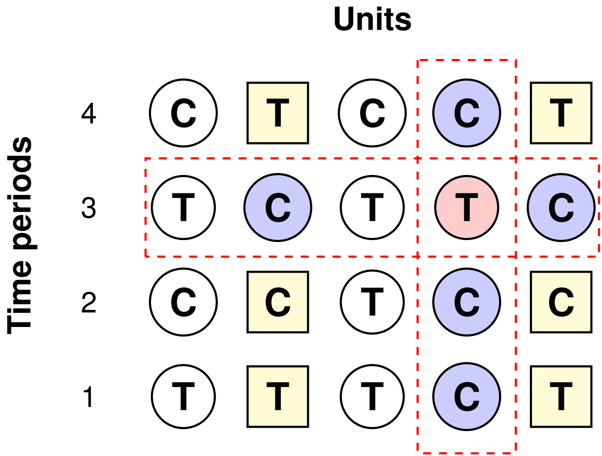
Matching and Two-way Fixed Effects Estimators

- Problem: No other unit shares the same unit and time



- Two kinds of mismatches
  1. Same treatment status
  2. Neither same unit nor same time

We Can Never Eliminate Mismatches

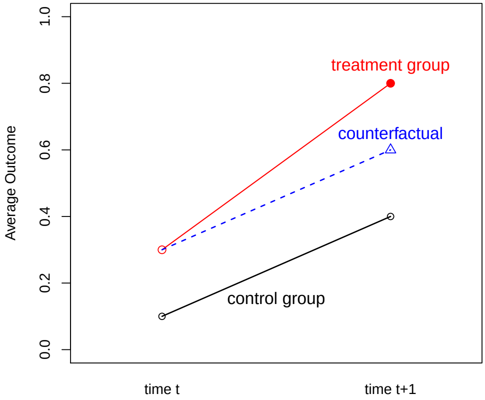


- To cancel time and unit effects, we must induce mismatches
- No weighted two-way fixed effects model eliminates mismatches

Difference-in-Differences Design

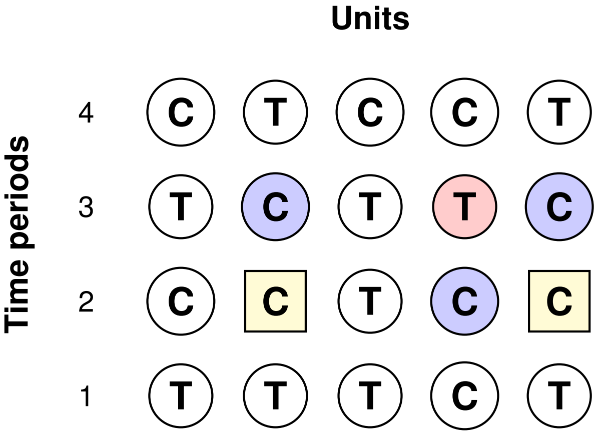
- Replace the model-based assumption with the design-based one
- Parallel trend assumption:

$$
\begin{aligned}
& \mathbb{E}(Y_{it}(0) - Y_{i,t-1}(0) \mid X_{it} = 1, X_{i,t-1} = 0) \\
= \; & \mathbb{E}(Y_{it}(0) - Y_{i,t-1}(0) \mid X_{it} = X_{i,t-1} = 0)
\end{aligned}
$$

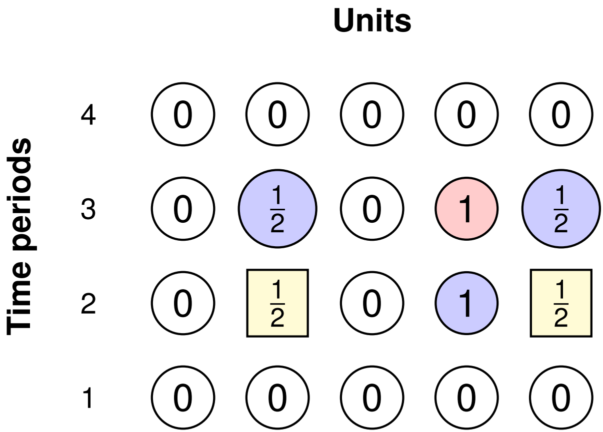


General DiD = Weighted Two-Way FE Effects

- $2 \times 2 \rightsquigarrow$ standard two-way fixed effects estimator works
- General setting: Multiple time periods, repeated treatments



- Regression weights:



- Weights can be negative $\Longrightarrow$ the method of moments estimator
- Fast computation is still available

Effects of GATT Membership on International Trade

1. Controversy
   - Rose (2004): No effect of GATT membership on trade
   - Tomz et al. (2007): Significant effect with non-member participants
2. The central role of fixed effects models:
   - Rose (2004): one-way (year) fixed effects for dyadic data
   - Tomz *et al.* (2007): two-way (year and dyad) fixed effects
   - Rose (2005): "I follow the profession in placing most confidence in the fixed effects estimators; I have no clear ranking between country-specific and country pair-specific effects."
   - Tomz *et al.* (2007): "We, too, prefer FE estimates over OLS on both theoretical and statistical ground"

Data and Methods

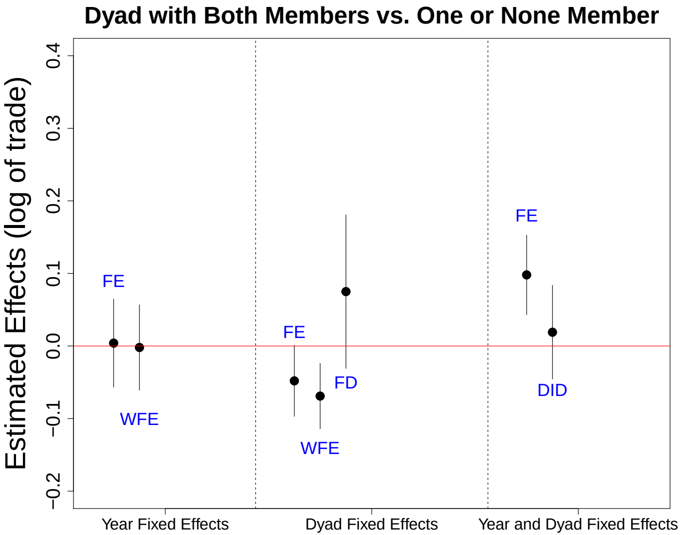
1. Data
   - Data set from Tomz et al. (2007)
   - Effect of GATT: 1948 – 1994
   - 162 countries, and 196,207 (dyad-year) observations
2. Year fixed effects model:

$$\ln Y_{it} = \alpha_t + \beta X_{it} + \delta^\top \mathbf{Z}_{it} + \epsilon_{it}$$

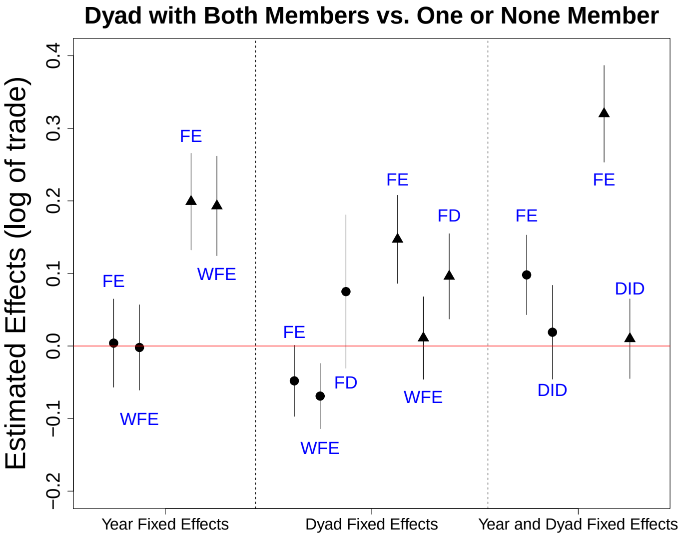
   - Y_{it}: trade volume
   - X_{it}: membership (formal/participants) Both vs. At most one
   - $\mathbf{Z}_{it}$: 15 dyad-varying covariates (e.g., log product GDP)
3. Weighted one-way fixed effects model:

$$\underset{(\alpha,\beta,\delta)}{\arg\min} \sum_{i=1}^{N} \sum_{t=1}^{T} W_{it}(\ln Y_{it} - \alpha_t - \beta X_{it} - \delta^\top Z_{it})^2$$

Empirical Results: Formal Membership



Empirical Results



Concluding Remarks

- Linear fixed effects models are attractive because they can adjust for unobserved confounders
- However, this advantage comes at costs
- Two key causal assumptions:
  1. treatments do not directly affect future outcomes
  2. outcomes do not directly affect future treatments and future time-varying covariates
- These assumptions can be relaxed under alternative selection-on-observables approaches
- Improve fixed effects estimators:
  1. Within-unit matching estimator $\rightsquigarrow$ no linearity assumption
  2. Design-based assumptions $\rightsquigarrow$ before-and-after, differnece-in-differences
  3. All of these can be written as weighted fixed effects regression
- R package **wfe** is available at CRAN

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More information about this and other research:

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