Causal Inference with Spatio-temporal Data: Estimating the Effects of Airstrikes on Insurgent Violence in Iraq

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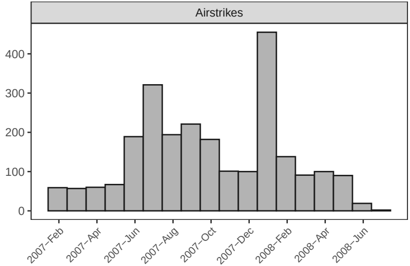
Motivation

- Increasing availability of unstructured data in social sciences
  - don't come in a nice matrix form $\leftrightsquigarrow$ survey, official statistics
  - text, images, audio, video, etc.
- How should we draw causal inference from these new types of data?
- Causal inference with spatio-temporal data
  - a time series of maps as data
  - treatment and outcome event locations in a continuous space
  - applications: crime incidents, disease outbreaks, etc.
- Methodological challenges
  1. spillover effects over space
  2. carryover effects over time
  3. infinitely many possible treatment and outcome locations
- Current practice
  1. arbitrary discretization of space
  2. assumptions about spillover and carryover effects

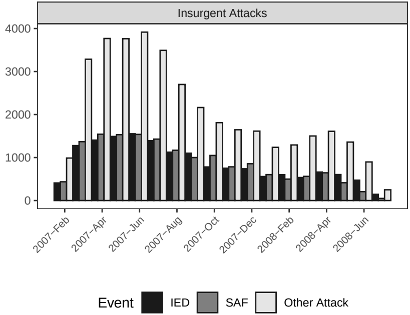
Impacts of Airstrikes on Insurgent Violence in Iraq

- Airstrikes as a principal tool for combating insurgency in civil wars
- Debate: whether or not airstrikes reduce subsequent insurgent attacks
  (e.g., Kocher *et al.* 2011; Dell and Querubin 2018; Lyall 2019; Mir and Moore 2019)
- Methodological limitations:
  - discretize continuous space into aggregate geographical units
  - simplifying assumptions about spillover and carryover effects
- American air campaign in Iraq:
  - declassified USAF data from Jan. 2007 to July 2008 ("surge" period)
  - daily data with precise geolocation for airstrikes and insurgent attacks
- Drivers of airstrikes:
  - prior patterns of insurgent attacks and airstrikes
  - presence of American forces
  - settlement patterns and road networks
  - economic aid
  - intelligence about high-value targets (small fraction)



(a) airstrikes over time

(b) Airstrikes over space



(c) Insurgent violence over time

(d) Insurgent violence over space

Contributions

- Causal inference with point process treatment and outcome
  - impossible to estimate causal effects of each treatment event
    - unrestricted spillover and carryover effects
    - probability of each treatment realization is zero $\rightsquigarrow$ lack of overlap
  - stochastic intervention based on the distribution of treatments
  - distribution of airstrikes as a military strategy
- Causal estimands under stochastic intervention
  - expected number of outcome events within a region of interest
  - various stochastic interventions
    1. change the dosage while keeping the distribution identical
    2. change the distribution while keeping the overall dosage constant
    3. intervention over multiple time periods
- The proposed IPW (inverse probability of treatment) estimator
  - overlap and unconfoundedness assumptions
  - consistency and asymptotic normality
- Simulation studies and empirical application

The Setup

- T time periods: $t = 1, 2, \ldots, T$
- Treatment variable
  - Ω: set of all possibly infinite locations that can receive the treatment
  - $W_t(s) \in \{0, 1\}$: binary treatment indicator for location s at time t
  - $W_t = \{W_t(s) : s \in \Omega\} \in \mathcal{W}$: treatment location map at time t
  - $S_{W_t} = \{s \in \Omega : W_t(s) = 1\}$: set of treatment-active locations at time t with $|S_{w_t}| < \infty$
  - $\overline{\boldsymbol{W}}_t = (W_1, W_2, \ldots, W_t)$: observed treatment history up to time t
- Outcome variable
  - $Y_t(s)$, Y_t, and $\overline{\boldsymbol{Y}}_t$ can be similarly defined
  - Potential outcome: $Y_t(\overline{\boldsymbol{w}}_t)$ where $w_t \in \mathcal{W}$ is a realized treatment and $\overline{\boldsymbol{w}}_t = (w_1, w_2, \ldots, w_t) \in \mathcal{W}^t$ is a treatment history realization at time t
  - Observed outcome: $Y_t = Y_t(\overline{\boldsymbol{W}}_t)$
  - $S_{Y_t(\overline{\boldsymbol{w}}_t)}$: set of outcome-active locations under treatment history $\overline{\boldsymbol{w}}_t$
  - History of all potential outcomes up to time t: $\overline{\mathcal{Y}}_t = \{Y_{t'}(\overline{\boldsymbol{w}}_{t'}) : \overline{\boldsymbol{w}}_{t'} \in \mathcal{W}^{t'}, t' \leq t\}$
- Time-varying confounders: X_t, $\overline{\mathbf{X}}_t$, $X_t(\overline{\boldsymbol{w}}_{t-1})$, and $\overline{\mathcal{X}}_t$

Stochastic Intervention

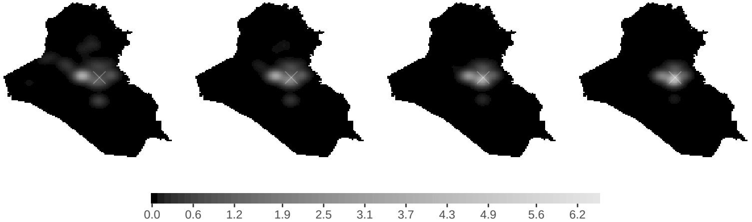
- Stochastic intervention: any distribution of treatment can be used
- We consider Poisson point process F_h
  - homogeneous Poisson point process with intensity h:
    For any disjoint region $B_1, B_2, \ldots, B_n \subset \Omega$, the number of events in each region B_i

$$N_{B_i}(W) \overset{\text{indep.}}{\sim} \text{Poisson}(h|B_i|)$$

  - non-homogeneous Poisson point process with intensity function $h(\omega)$:

$$N_{B_i}(W) \overset{\text{indep.}}{\sim} \text{Poisson}\left(\int_{B_i} h(\omega)d\omega\right)$$

- Example:



Causal Estimands

- Expected number of outcome-active locations in region B at time t under stochastic intervention F_h conducted at time t

$$\overline{N}_{Bt}(F_h) \;=\; \int_{\mathcal{W}} N_B(Y_t(\overline{\boldsymbol{W}}_{t-1}, w_t))dF_h(w_t)$$

- Further average this quantity over time:

$$\overline{N}_B(F_h) \;=\; \frac{1}{T}\sum_{t=1}^{T} \overline{N}_{Bt}(F_h)$$

- We can compare the different interventions:

$$\tau_B(F_{h'}, F_h) \;=\; \overline{N}_B(F_{h'}) - \overline{N}_B(F_h)$$

Stochastic Intervention over Multiple Time Periods

- Consider a non-dynamic stochastic intervention over M time periods

$$F_{\boldsymbol{h}} \;=\; F_{h_1} \times \cdots \times F_{h_M} \quad \text{where } \boldsymbol{h} = (h_1, h_2, \ldots, h_M)$$

- Expected number of outcome-active locations in region B at time t under stochastic intervention $F_{\boldsymbol{h}}$ conducted from time $t - M + 1$ to t

$$\overline{N}_{Bt}(F_{\boldsymbol{h}}) \;=\; \int_{\mathcal{W}} \cdots \int_{\mathcal{W}} N_B(Y_t(\overline{\boldsymbol{W}}_{t-M}, w_{t-M+1}, \ldots, w_t)) \\ dF_{h_M}(w_{t-M+1}) \cdots dF_{h_1}(w_t)$$

- Average this quantity over time:

$$\overline{N}_B(F_{\boldsymbol{h}}) \;=\; \frac{1}{T - M + 1} \sum_{t=M}^{T} \overline{N}_{Bt}(F_{\boldsymbol{h}})$$

- Comparison of different interventions:

$$\tau_B(F_{\boldsymbol{h}'}, F_{\boldsymbol{h}}) \;=\; \overline{N}_B(F_{\boldsymbol{h}'}) - \overline{N}_B(F_{\boldsymbol{h}})$$

e.g., lagged effects with $h_M \neq h'_M$ and $\boldsymbol{h}_{-M} = \boldsymbol{h}'_{-M}$

Assumptions

1. Unconfoundedness: treatment is independent of all potential (past and future) paths for the outcome and time-varying confounders conditional on the observed history

$$f(W_t \mid \overline{\boldsymbol{W}}_{t-1}, \overline{\boldsymbol{Y}}_{t-1}, \overline{\boldsymbol{X}}_t, \{\overline{\mathcal{Y}}_T, \overline{\mathcal{X}}_T\}) \ = \ f(W_t \mid \overline{\boldsymbol{W}}_{t-1}, \overline{\boldsymbol{Y}}_{t-1}, \overline{\boldsymbol{X}}_t)$$

$\rightsquigarrow$ the generalization of the non-anticipating assumption for time-series experiments (Bojinov and Shephard, 2019)

2. Overlap: there exists a constant $\delta_W > 0$ such that

$$\underbrace{f(W_t = w \mid \overline{\boldsymbol{W}}_{t-1}, \overline{\boldsymbol{Y}}_{t-1}, \overline{\boldsymbol{X}}_t)}_{propensity \ \ score} \ > \ \delta_W \cdot \underbrace{f_h(w)}_{density \ \ of \ \ F_h} \quad \text{for all } w \in \mathcal{W}$$

$\rightsquigarrow$ the ratio $f_h(w)/f(W_t = w \mid \overline{\boldsymbol{W}}_{t-1}, \overline{\boldsymbol{Y}}_{t-1}, \overline{\boldsymbol{X}}_t)$ is bounded

The Proposed Estimator

- Inverse probability of treatment weighting (IPW)
- Kernel smoothing of spatial point patterns
- Estimated outcome surface at $\omega \in \Omega$

$$\widehat{Y}_t(F_h;\omega) \; = \; \underbrace{\frac{\overbrace{f_h(W_t)}^{counterfactual \;\; distribution}}{\hat{f}(W_t \mid \overline{\boldsymbol{W}}_{t-1}, \overline{\boldsymbol{Y}}_{t-1}, \overline{\boldsymbol{X}}_t)}}_{actual \;\; distribution} \;\; \underbrace{\sum_{s \in S_{Y_t}} K_b(\|\omega - s\|)}_{spatially \;\; smoothed \;\; outcome}$$

where K_b is the scaled Kernel function with bandwidth parameter b

- Estimated number of outcome-active locations in region B

$$\widehat{\widehat{N}}_{Bt}(F_h) \; = \; \int_B \widehat{Y}_t(F_h;\omega) d\omega$$

- Averaging over time

$$\widehat{\widehat{N}}_B(F_h) \; = \; \frac{1}{T} \sum_{t=1}^{T} \widehat{\widehat{N}}_{Bt}(F_h)$$

Estimation for Intervention over Multiple Time Periods M

- Estimated outcome surface at $\omega \in \Omega$

$$\widehat{Y}_t(F_{\boldsymbol{h}}; \omega) \; = \; \underbrace{\prod_{j=t-M+1}^{t} \frac{f_{h_{t-j+1}}(W_j)}{\hat{f}(W_j \mid \overline{\boldsymbol{W}}_{j-1}, \overline{\boldsymbol{Y}}_{j-1}, \overline{\boldsymbol{X}}_j)}}_{\text{product of } M \text{ ratios}} \sum_{s \in S_{Y_t}} K_b(\|\omega - s\|)$$

where K_b is the scaled Kernel function with bandwidth parameter b

- Estimated number of outcome-active locations in region B

$$\widehat{\widehat{N}}_{Bt}(F_{\boldsymbol{h}}) \; = \; \int_B \widehat{Y}_t(F_{\boldsymbol{h}}; \omega) d\omega$$

- Averaging over time

$$\widehat{\widehat{N}}_B(F_{\boldsymbol{h}}) \; = \; \frac{1}{T - M + 1} \sum_{t=M}^{T} \widehat{\widehat{N}}_{Bt}(F_{\boldsymbol{h}})$$

Asymptotic Normality and Variance Estimation

- Suppose that there exists v such that

$$\frac{1}{T-M+1}\sum_{t=M}^{T} v_t \xrightarrow{p} v \quad \text{as } T \longrightarrow \infty$$

where $v_t = \mathbb{V}\left(\widehat{\overline{N}}_{Bt}(F_{\boldsymbol{h}}) \mid \overline{\boldsymbol{W}}_{t-M}, \overline{\mathcal{Y}}_T, \overline{\mathcal{X}}_T\right)$.

- Then, under some regularity conditions, we have,

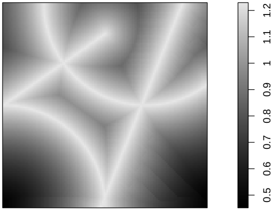
$$\sqrt{T}\left(\widehat{\overline{N}}_{B}(F_{\boldsymbol{h}}) - \overline{N}_{B}(F_{\boldsymbol{h}})\right) \xrightarrow{d} \mathcal{N}(0, v)$$

where the proof is based on the martingale theory

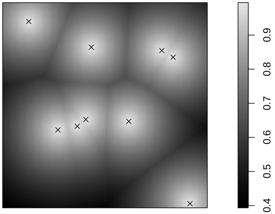
- Time-specific variance v_t cannot be estimated
- We use the upper bound $v_t \leq \mathbb{E}\left(\widehat{\overline{N}}_{Bt}(F_{\boldsymbol{h}})^2 \mid \overline{\boldsymbol{W}}_{t-M}, \overline{\mathcal{Y}}_T, \overline{\mathcal{X}}_T\right)$
- Estimated propensity score $\rightsquigarrow$ smaller variance

Simulation Studies: Setup

- Times series of length $T = 200, 400, 500$ on unit square $\Omega = [0,1]^2$
- Two time-invariant confounders $X^1(\omega), X^2(\omega)$:
  1. draw hypothetical road networks using lines and arcs
  2. confounder as exponential decay of distance to the closest line (arc)
- Two time-varying confounders $X^3(\omega), X^4(\omega)$:
  1. draw points from non-homogeneous Poisson process based on $X^1(\omega)$
  2. confounder as exponential decay of distance to the closest point



(a) Time-invariant confounder $X^1(\omega)$



(b) Time-varying confounder $X^3(\omega)$

- Treatment assignment: non-homogeneous Poisson point process

$$\lambda_t^W(\omega) \; = \; \exp\{\alpha_0 + \boldsymbol{\alpha}_{\mathbf{X}}^\top \mathbf{X}_t(\omega) + \alpha_W W_{t-1}^*(\omega) + \alpha_Y Y_{t-1}^*(\omega)\}$$

  where $W_{t-1}^*(\omega) = \exp\{-2D_W(\omega)\}$ and $Y_{t-1}^*(\omega) = \exp\{-2D_Y(\omega)\}$ with D_W and D_Y being the distance from ω to the closest treatment and outcome active locations at $t-1$

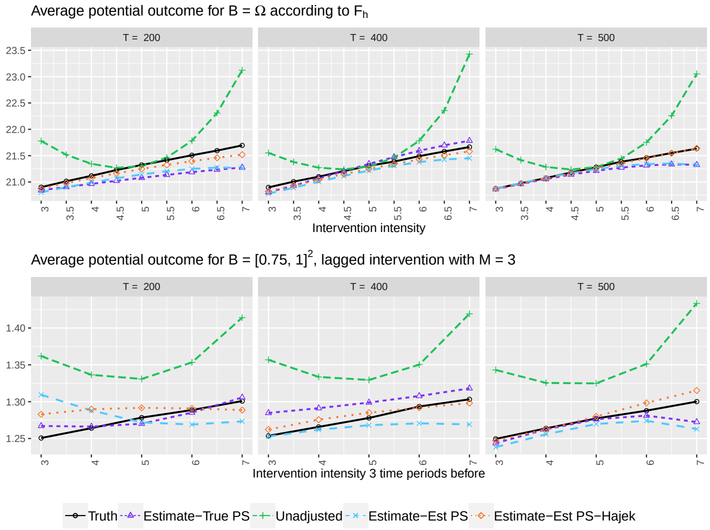
- Outcome: non-homogeneous Poisson point process

$$\lambda_t^Y(\omega) \; = \; \exp\{\gamma_0 + \boldsymbol{\gamma}_{\mathbf{X}}^\top \mathbf{X}_t(\omega) + \gamma_W W_{(t-3):t}^*(\omega) + \gamma_Y Y_{t-1}^*(\omega)\}$$

  where $W_{(t-3):t}^*(\omega) = \exp\{-2D_W^*(\omega)\}$ with D_w^* being the distance from ω to the closest treatment active locations from time $t-3$ to t

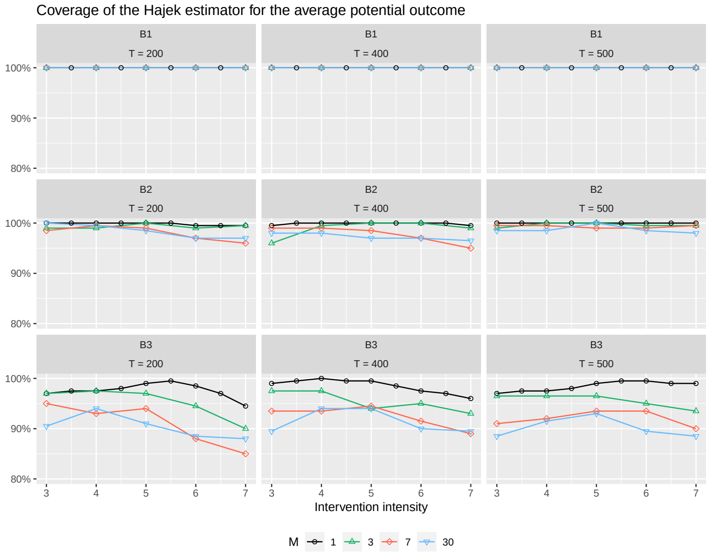
- Stochastic interventions: homogeneous Poisson point process
  1. $F_{\boldsymbol{h}} = F_h^M$ with intensity $h = 3, \ldots, 7$ with $M = 1, 3, 7, 30$
  2. $F_{\boldsymbol{h}} = F_{h_1} \times F_{h_2} \times F_{h_3}$ where $h_3 = 3, \ldots, 7$ and $h_1 = h_2 = 5$

Simulation Results



- Estimation improves as T increases and worsens as h increases
- Hájek adjustment substantially improves the performance

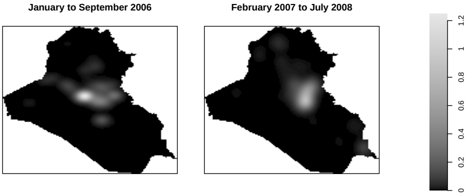
Coverage of Hájek Estimator's 95% Confidence Intervals



- Performance improves as T increases and worsens as M increases

Empirical Analysis: Setup

- Estimate the baseline treatment distribution $\phi_0(\omega)$ based on the airstrikes data from January to September, 2006



1. How does increasing airstrikes affect insurgent violence?
   $\rightsquigarrow$ vary $c > 0$ for $h(\omega) = c \cdot \phi_0(\omega)$
2. How long does it take for these effects to be realized?
   $\rightsquigarrow$ vary c for $h_M(\omega) = c \cdot \phi_0(\omega)$ and $h_1(\omega) = \cdots = h_{M-1}(\omega) = \phi_0(\omega)$
3. How does the shift in the prioritization of certain locations for airstrikes change the spatial pattern of insurgent attacks?
   $\rightsquigarrow$ vary $\alpha > 0$ for $h_\alpha(\omega) = c_\alpha \cdot \phi_0(\omega) d_\alpha(\omega)$ with $\int_\Omega h_\alpha(\omega) d\omega = c$
   - power density $d_\alpha(\omega) \propto d(\omega)^\alpha$
   - $d(\omega)$ = the normal density centered at s_f with precision α

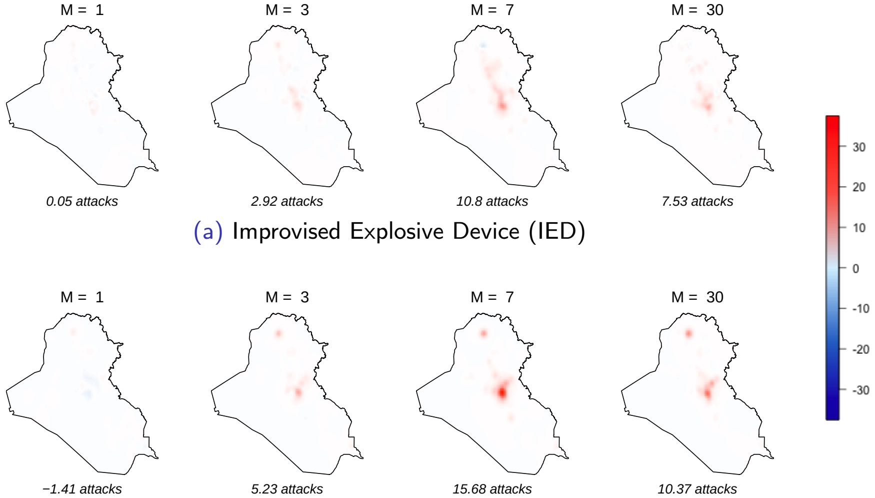
Propensity Score Model Specification

- Non-homogeneous Poisson point process
  - prior airstrikes over the last day, week, and month

$$\overline{W}^*_{t-1}(\omega) = \sum_{j=1}^{7} \sum_{s \in S_{W_{t-j}}} \exp\{-\|s - \omega\|\}$$

  - prior insurgent attacks over the last day, week, and month
  - prior show-of-force over the last day, week, and month
  - amount of US aid in each district over the past month
  - distances from major cities, road networks, rivers, and settlements
  - log population of governorate (measured in 2003), temporal splines

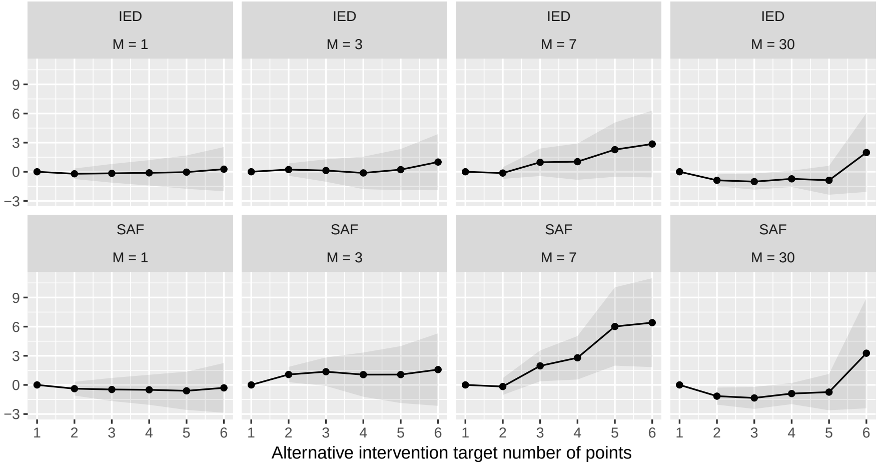
Increasing the Expected Number of Airstrikes from 1 to 6 per Day Leads to More Insurgent Attacks



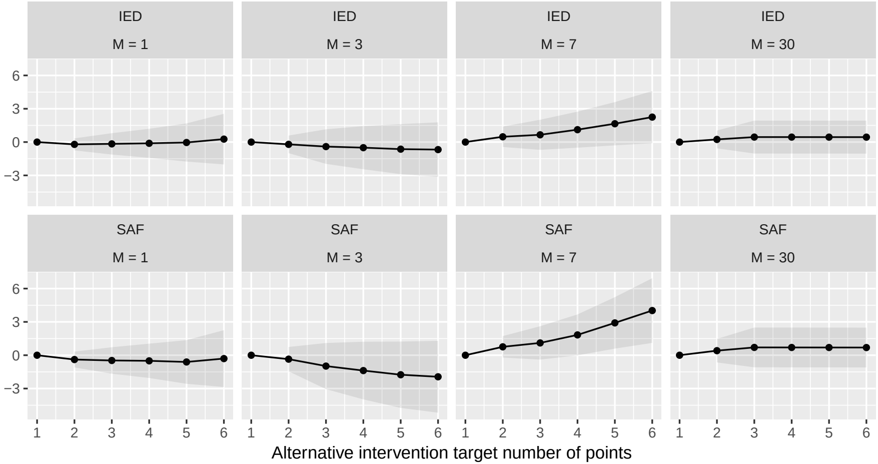
(a) Improvised Explosive Device (IED)

(b) Small Armed Fire (SAF)

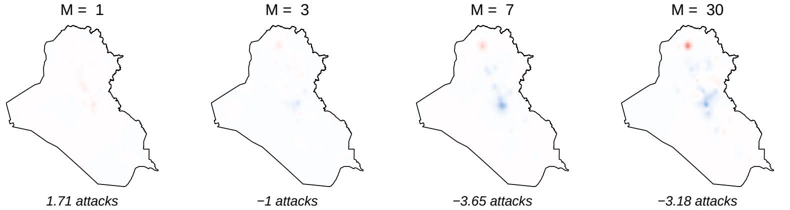
Effect of Increasing the Airstrikes **for *M* Days** on the Number of Insurgent Attacks within Baghdad



Effect of Increasing the Airstrikes *M* **Days Ago** on the Number of Insurgent Attacks within Baghdad



Increasing the Priority of Baghdad as a Focal Point of Airstrikes Shifts Attacks to Mosul when *M* is Large



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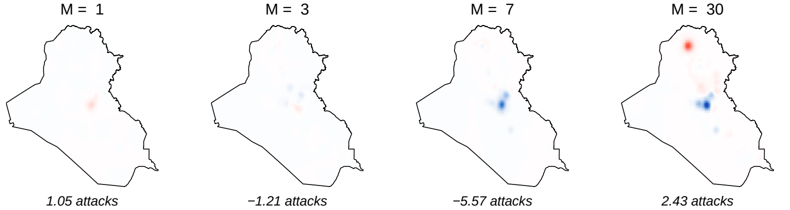
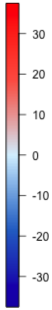


(b) Small Armed Fire (SAF)



Concluding Remarks

- A new approach to causal inference with spatio-temporal data
  - directly model point patterns without arbitrary aggregation
  - allow for unstructured spillover and carryover effects
- Key idea: stochastic intervention
  - consider treatment distributions rather than fixed treatment values
  - can handle infinitely many possible treatment locations
  - combine this with spatial smoothing for outcome point process
- Effects of airstrikes on insurgent attacks in Iraq
  - airstrike strategies as stochastic interventions
  - flexible estimation of spillover and carryover effects
- Future research:
  - causal inference with unstructured data such as texts
  - civilian casualty as mediator; comparison with hearts and minds
- Paper at https://imai.fas.harvard.edu/research/spatiotempo.html