Causal Mediation Analysis in **R**

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June 18, 2009

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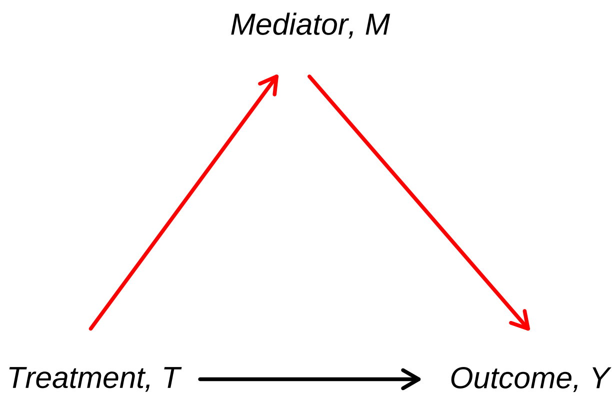
Quantitative Methodology and Causal Mechanisms

- Investigation of causal mechanisms via intermediate variables
- Randomized experiments can only determine *whether* the treatment causes changes in the outcome
- Not *how* and *why* the treatment affects the outcome
- Social scientists use qualitative methods (e.g. process tracing) to answer these questions
- How can quantitative research be used to identify causal mechanisms?

Causal Mediation Analysis



- Quantities of interest: Direct and indirect effects
- Traditional tools: Path analysis, structural equation modeling
- Fast growing methodological literature

Common Practice

- Regression

$$Y_i = \alpha + \beta T_i + \gamma M_i + \delta X_i + \epsilon_i$$

- Each coefficient is interpreted as a causal effect
- Sometimes, it's called marginal effect
- Idea: increase T_i by one unit while holding M_i and X_i constant

- The Problem: Post-treatment bias
- If you change T_i, that may also change M_i
- Usual advice: only include causally prior (or pre-treatment) variables
- But, then you lose causal mechanisms!

Defining Causal Mediation Effects

- Binary treatment (can be generalized): $T_i \in \{0, 1\}$
- Mediator: $M_i \in \mathcal{M}$
- Outcome: $Y_i \in \mathcal{Y}$
- Observed covariates: $X_i \in \mathcal{X}$

- Potential mediators: $M_i(t)$ where $M_i = M_i(T_i)$
- Potential outcomes: $Y_i(t, m)$ where $Y_i = Y_i(T_i, M_i(T_i))$

- Total causal effect: $\tau_i \equiv Y_i(1, M_i(1)) - Y_i(0, M_i(0))$
- Causal mediation effects: $\delta_i(t) \equiv Y_i(t, M_i(1)) - Y_i(t, M_i(0))$
- Direct effects: $\zeta_i(t) \equiv Y_i(1, M_i(t)) - Y_i(0, M_i(t))$

- Total effect = Mediation (indirect) effect + Direct effect:

$$\tau_i = \delta_i(t) + \zeta_i(1-t) = \frac{1}{2}\sum_{t=0}^{1}\{\delta_i(t) + \zeta_i(t)\}$$

Interpreting Causal Mediation Effects

- $\delta_i(t)$: Causal effect of the change in M_i on Y_i that would be induced by T_i, holding actual treatment constant at t
- $\zeta_i(t)$: Causal effect of T_i on Y_i, holding mediator constant at its potential value that would realize when $T_i = t$

- Different from *controlled direct effects*: $Y_i(t, m) - Y_i(t, m')$

- Mediation effects — identify causal paths from T_i to Y_i
- Controlled effects — study how T_i moderates the effect of M_i on Y_i

- Average Causal Mediation Effects:

$$\bar{\delta}(t) \equiv \mathbb{E}(\delta_i(t)) = \mathbb{E}\{Y_i(t, M_i(1)) - Y_i(t, M_i(0))\}$$

Nonparametric Identification

- Problem: $Y_i(t, M_i(t))$ is observed but $Y_i(t, M_i(1-t))$ can *never* be observed
- Proposed identification assumption: Sequential Ignorability

$$\{Y_i(t', m), M_i(t)\} \perp\!\!\!\perp T_i \mid X_i = x,$$

$$Y_i(t', m) \perp\!\!\!\perp M_i \mid T_i = t, X_i = x$$

Theorem 1 (Imai, Keele, and Yamamoto (2008))

Under sequential ignorability,

$$\bar{\delta}(t) = \int\int \mathbb{E}(Y_i \mid M_i, T_i = t, X_i)\{dP(M_i \mid T_i = 1, X_i) - dP(M_i \mid T_i = 0, X_i)\}\, dP(X_i),$$

$$\bar{\zeta}(t) = \int\int \{\mathbb{E}(Y_i \mid M_i, T_i = 1, X_i) - \mathbb{E}(Y_i \mid M_i, T_i = 0, X_i)\}\, dP(M_i \mid T_i = t, X_i)\, dP(X_i).$$

Inference Under Sequential Ignorability

- Model outcome and mediator
- Outcome model: $p(Y_i \mid T_i, M_i, X_i)$
- Mediator model: $p(M_i \mid T_i, X_i)$
- Can use parametric or nonparametric regressions; probit, logit, GAM, quantile regression etc.
- Two new algorithms for statistical inference:
  1. Quasi-Bayesian approximation: approximating the posterior by the sampling distribution of MLE
  2. Bootstrap: works for nonparametric models as well as parametric ones
- The details and examples are in Imai, Keele and Tingley (2009)

Need for Sensitivity Analysis

- The sequential ignorability assumption is often too strong
- Need to assess the robustness of findings via sensitivity analysis
- Question: How large a departure from the key assumption must occur for the conclusions to no longer hold?

- Parametric sensitivity analysis by assuming

$$\{Y_i(t', m), M_i(t)\} \perp\!\!\!\perp T_i \mid X_i = x$$

but not

$$Y_i(t', m) \perp\!\!\!\perp M_i \mid T_i = t, X_i = x$$

- Possible existence of unobserved *pre-treatment* confounder

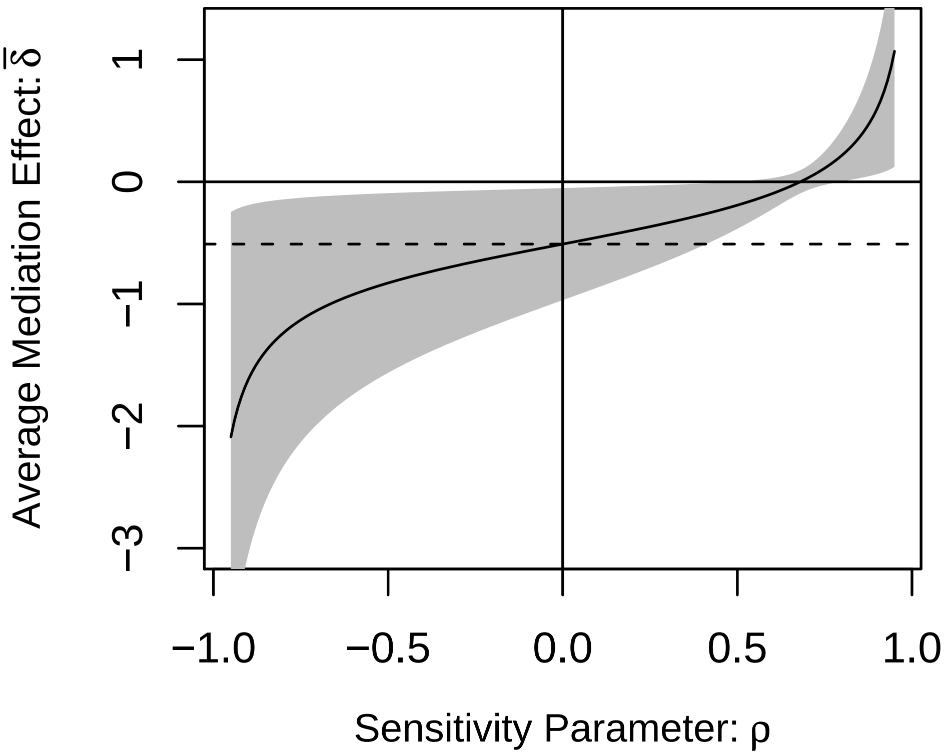
Parametric Sensitivity Analysis

- Consider LSEM (aka Baron-Kenny procedure):

$$\begin{aligned} M_i &= \alpha_2 + \beta_2 T_i + \epsilon_{2i}, \\ Y_i &= \alpha_3 + \beta_3 T_i + \gamma M_i + \epsilon_{3i}. \end{aligned}$$

- Sensitivity parameter: $\rho \equiv \mathrm{Corr}(\epsilon_{2i}, \epsilon_{3i})$
- Sequential ignorability implies $\rho = 0$
- Set ρ to different values and see how mediation effects change
- An alternative explanation of ρ based on R^2
- Work for probit models – binary outcome, binary mediator, etc.
- Difficult to construct a more general sensitivity analysis

An Example Sensitivity Analysis Plot



Facilitating Interpretation

- How big is ρ?
- An unobserved (pre-treatment) confounder formulation:

$$\epsilon_{2i} \quad = \quad \lambda_2 U_i + \epsilon'_{2i} \quad \text{and} \quad \epsilon_{3i} \quad = \quad \lambda_3 U_i + \epsilon'_{3i},$$

- Assume $Y_i(t', m) \perp\!\!\!\perp M_i \mid T_i = t, U_i = u$
- Assume also $\epsilon'_{2i} \perp\!\!\!\perp U_i$ and $\epsilon'_{3i} \perp\!\!\!\perp U_i$
- Proportion of previously unexplained variance explained by the unobserved confounder

$$R_M^{2*} \equiv 1 - \frac{\mathrm{var}(\epsilon'_{2i})}{\mathrm{var}(\epsilon_{2i})} \quad \text{and} \quad R_Y^{2*} \equiv 1 - \frac{\mathrm{var}(\epsilon'_{3i})}{\mathrm{var}(\epsilon_{3i})}$$

- Proportion of original variance explained by the unobserved confounder

$$\widetilde{R}_M^2 \equiv \frac{\mathrm{var}(\epsilon_{2i}) - \mathrm{var}(\epsilon'_{2i})}{\mathrm{var}(M_i)} \quad \text{and} \quad \widetilde{R}_Y^2 \equiv \frac{\mathrm{var}(\epsilon_{3i}) - \mathrm{var}(\epsilon'_{3i})}{\mathrm{var}(Y_i)}$$

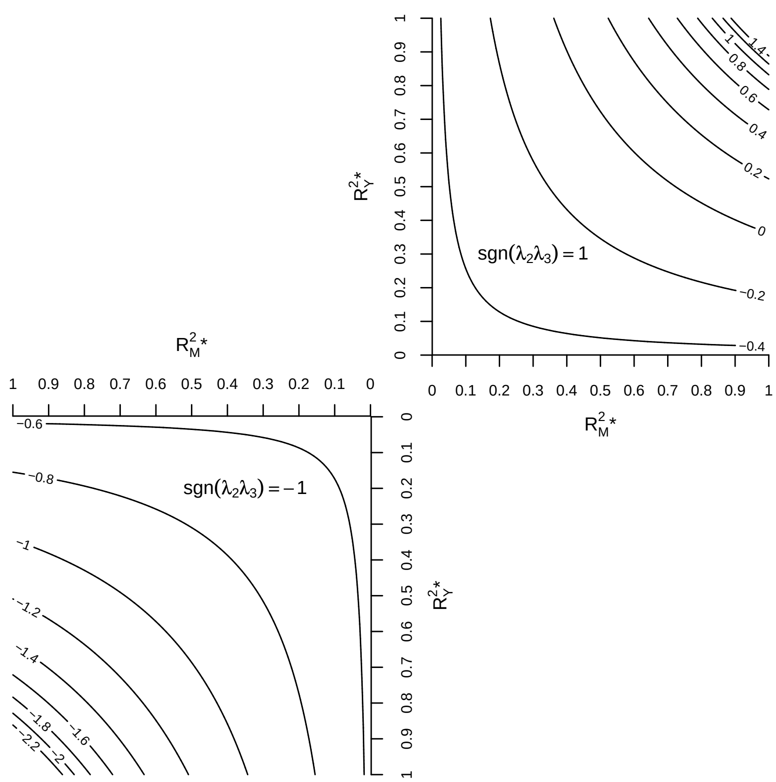
- Specify $\mathrm{sgn}(\lambda_2\lambda_2)$ and ${R_M^*}^2, {R_Y^*}^2$ (or $\widetilde{R}_M^2, \widetilde{R}_Y^2$)

$$\rho = \mathrm{sgn}(\lambda_2\lambda_3) R_M^* R_Y^* = \frac{\mathrm{sgn}(\lambda_2\lambda_3)\widetilde{R}_M \widetilde{R}_Y}{\sqrt{(1-R_M^2)(1-R_Y^2)}},$$

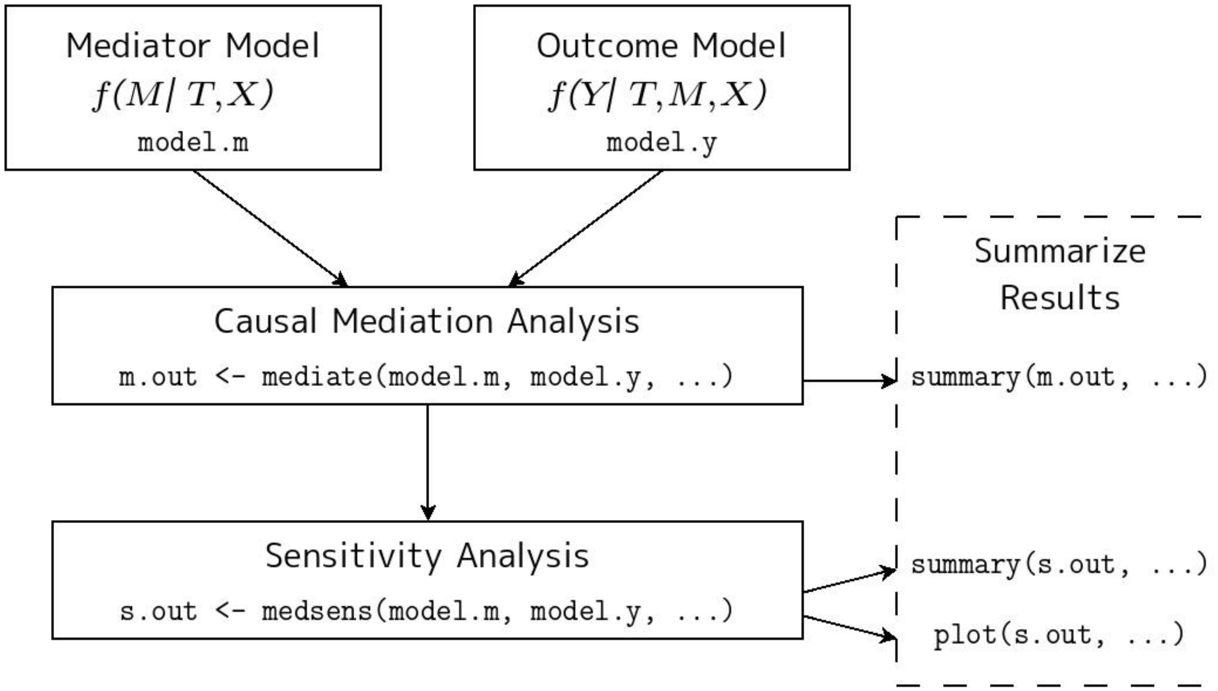
where R_M^2 and R_Y^2 are based on

$$\begin{aligned} M_i &= \alpha_2 + \beta_2 T_i + \epsilon_{2i} \\ Y_i &= \alpha_3 + \beta_3 T_i + \gamma M_i + \epsilon_{3i} \end{aligned}$$

Proportion of unexplained variance explained by an unobserved confounder



Overview of **R** Package **mediation**



- Object-oriented nature of **R** made this design possible

An Illustrative Example

- Job Search Intervention Study (JOBS II)
- A randomized evaluation of a job training program
- Treatment: Job-skills workshop
- Mediator: a continuous measure of job-search self-efficacy
- Outcome: a binary measure of employment
- Question: Does the workshop improve the prospect of future employment by increasing the level of job-search self-efficacy?

Step 1: Fitting the Outcome and Mediator Models

```
> # load the library
> library(mediation)
> # load the data set
> data(jobs)
>
> # fit the mediator model
> model.m <- lm(job_seek ~ treat + depress1 +
     econ_hard + sex + age + occp + marital +
     nonwhite + educ + income, data = jobs)
>
> # fit the outcome model
> model.y <- glm(work1 ~ treat + job_seek +
     depress1 + econ_hard + sex + age + occp +
     marital + nonwhite + educ + income,
     family = binomial(link="probit"), data = jobs)
```


Step 2: Conducting Causal Mediation Analysis

```
> # mediation analysis
> m.out <- mediate(model.m, model.y, sims = 1000,
    T = "treat", M = "job_seek")
> # summary of the analysis
> summary(m.out)

 Causal Mediation Analysis

Quasi-Bayesian Confidence Intervals

Mediation Effect: 0.003558 95% CI -0.001074 0.010679
Direct Effect: 0.05455 95% CI -0.006838 0.116466
Total Effect: 0.0581 95% CI -0.003083 0.119178
Proportion of Total Effect via Mediation:
              0.05687 95% CI -0.2028 0.4490
```

Step 3: Conducting Sensitivity Analysis

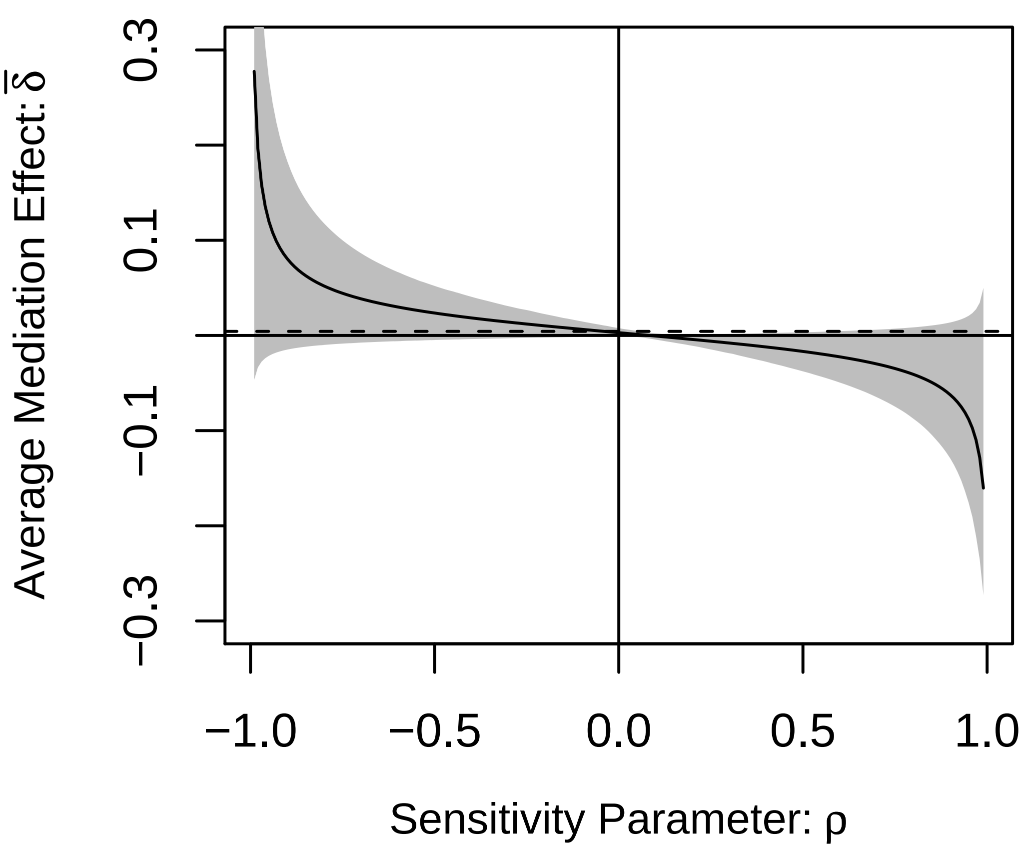
```
> s.out <- medsens(model.m, model.y, sims = 1000,
    T = "treat", M = "job_seek", INT = FALSE,
    DETAIL=FALSE)
> summary(s.out)

Mediation Sensitivity Analysis

Sensitivity Region

       Rho Med. Eff. 95% CI Lower 95% CI Upper
 [1,] -0.9    0.1183      -0.0274       0.2640
 [2,] -0.8    0.0715      -0.0166       0.1597
 [3,] -0.7    0.0489      -0.0115       0.1093
... output truncated

> plot(s.cont)
```

Concluding Remarks

- Quantitative analysis can be used to identify causal mechanisms!
- Wide applications in many social scientific disciplines
- Sensitivity analysis is critical

- Development of easy-to-use software **mediation**
- Object-oriented nature of **R** facilitated this development
- Future extensions: multiple mediators, sensitivity analysis for other models

Papers and Software

- Keele, Tingley, Yamamoto, and Imai. (2009). **mediation**: R Package for Causal Mediation Analysis. available at CRAN
- Imai, Keele, Tingley, and Yamamoto. (2009). "Causal Mediation Analysis in **R**."

- Imai, Keele, and Yamamoto. (2008). "Identification, Inference, and Sensitivity Analysis for Causal Mediation Effects."
- Imai, Keele, and Tingley. (2009). "A General Approach to Causal Mediation Analysis."

- All are available at **http://imai.princeton.edu/**