# A Statistical Method for Empirical Testing of Competing Theories

**Kosuke Imai** — Princeton University

**Dustin Tingley** — Harvard University

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# Motivation

- Empirical testing of competing theories lies at the heart of social science research
- Need to test the validity of alternative theories explaining the same phenomena
- "theory confirmation is not possible when a theory is tested in isolation, regardless of the statistical approach" (Clarke)

- Common statistical methods used in the discipline:
  1. "Garbage-can" regressions: atheoretical (Achen)
  2. Model selection methods (e.g., AIC, BIC, Vuong test, $J$ test): All or nothing, Independence of Irrelevant Alternatives (IIA)
- Key distinction between causal and predictive inference

# The Proposed Approach

- Theoretical heterogeneity: No single theory can explain everything
- Explaining when each theory "works"
  1. Testing the entire theory including its assumptions rather than just its implications
  2. Leading to further theory development
- Finite mixture models
  1. A well-known, very general class of statistical models
  2. Can test more than two theories at the same time
  3. Under-utilized in political science except a few studies
- Quantities of interest:
  1. population proportion of observations consistent with each theory
  2. how this proportion varies as a function of observed characteristics
  3. probability that a particular observation is consistent with a theory
  4. list of observations that are consistent with each theory

# An Example: Determinants of Trade Policies

- Hiscox (2002, *APSR*) analyzes US legislative voting on trade bills
- Stolper-Samuelson (SS) model: cleavages along factoral lines
  - The highly skilled favor liberalization while the low-skilled oppose it
- Ricardo-Viner (RV) model: cleavages along sectoral lines
  - Exporters favor liberalization while importers oppose it
- Key contribution: the applicability of the two models depends on the level of factor mobility in the US economy
  - If capital is highly mobile across industries, then the conditions for the SS model are satisfied
  - If capital is highly specific, then the conditions for the RV model are satisfied

# Finite Mixture Models: A Review

- $M$ competing theories, each of which implies a statistical model $f_m(y \mid x)$ for $m = 1, \ldots, M$
- The data generating process:

$$Y_i \mid X_i, Z_i \sim f_{Z_i}(Y_i \mid X_i, \theta_{Z_i})$$

  where $Z_i$ is the *latent* variable indicating the theory which generates observation $i$
- The observed-data likelihood function:

$$L_{obs}(\Theta, \Pi \mid \{X_i, Y_i\}_{i=1}^N) = \prod_{i=1}^N \left\{ \sum_{m=1}^M \pi_m f_m(Y_i \mid X_i, \theta_m) \right\},$$

  where $\pi_m = \Pr(Z_i = m)$ is the population proportion of observations generated by theory $m$
- $\pi_m$: a measure of overall performance of the theory

- Explaining theoretical heterogeneity:

$$\Pr(Z_i = m \mid W_i) \quad = \quad \pi_m(W_i, \psi_m),$$

- Predicting which theory has generated a particular observation:

$$\begin{aligned} \zeta_{i,m} &= \Pr(Z_i = m \mid \Theta, \Pi, \{X_i, Y_i\}_{i=1}^N) \\ &= \frac{\pi_m f_m(Y_i \mid X_i, \theta_m)}{\sum_{m'=1}^M \pi_{m'} f_{m'}(Y_i \mid X_i, \theta_{m'})} \end{aligned}$$

- Grouped observations:

$$\zeta_{i,m} \quad = \quad \frac{\pi_m \prod_{j=1}^{J_i} f_m(Y_{ij} \mid X_{ij}, \theta_m)}{\sum_{m'=1}^M \pi_{m'} \prod_{j=1}^{J_i} f_{m'}(Y_{ij} \mid X_{ij}, \theta_{m'})}$$

- Estimation: Expectation-Maximization or Markov chain Monte Carlo algorithm
- Implementation: `flexmix` package in R by Leisch and Gruen

# Statistically Significantly Consistent with a Theory

- Identification of observations that are statistically significantly consistent with each theory
- Idea: If $\zeta_{i,m}$ is greater than a threshold $\lambda_m$, then include observation $i$ in the list

- Problem of multiple testing: false positives
- Simple example:
  - 10 Independent 0.05 level tests
  - $1 - 0.95^{10} \approx 0.4$ chance of at least one false discovery

- Solution: choose the smallest value of $\lambda_m$ such that the posterior expected value of false discovery rate on the resulting list does not exceed a prespecified threshold $\alpha_m$:

$$\lambda_m^* = \inf\left\{\lambda_m : \frac{\sum_{i=1}^N (1-\hat{\zeta}_{i,m})\mathbf{1}\{\hat{\zeta}_{i,m} \geq \lambda_m\}}{\sum_{i=1}^N \mathbf{1}\{\hat{\zeta}_{i,m} \geq \lambda_m\} + \prod_{i=1}^N \mathbf{1}\{\hat{\zeta}_{i,m} < \lambda_m\}} \leq \alpha_m\right\}$$

# Measuring the Overall Performance of a Theory

1. Population proportion of observations consistent with each theory: $\pi_m$ or $\sum_{i=1}^{N} \hat{\zeta}_{i,m}/N$
2. Sample proportion of the observations statistically significantly consistent with the theory

# Testing the Competing Theories of Trade Policy

- Data
  - Congressional voting data on 55 trade bills spanning over 150 years
  - A combined measure of factor specificity for a given year
  - State-level measures of relevant covariates for each model
- The original analysis used the $J$ test in logistic regression with bill fixed effects
- The $J$ test in its original form:

$$Y_i = (1-\pi) f(X_i, \beta) + \pi g(X_i, \gamma) + \epsilon_i,$$

  - The null hypothesis, $Y_i = f(X_i, \beta) + \epsilon_i$
  - The alternative hypothesis, $Y_i = g(X_i, \gamma) + \epsilon_i$
- Finite mixture models do not assume $\pi$ is either 0 or 1

# The Mixture Model Specification

- Assuming all votes for the same bill belong to the same model
- Stolper-Samuelson Model:

$$\text{logit}^{-1}(\beta_0 + \beta_1 \texttt{profit}_{ij} + \beta_2 \texttt{manufacture}_{ij} + \beta_3 \texttt{farm}_{ij})$$

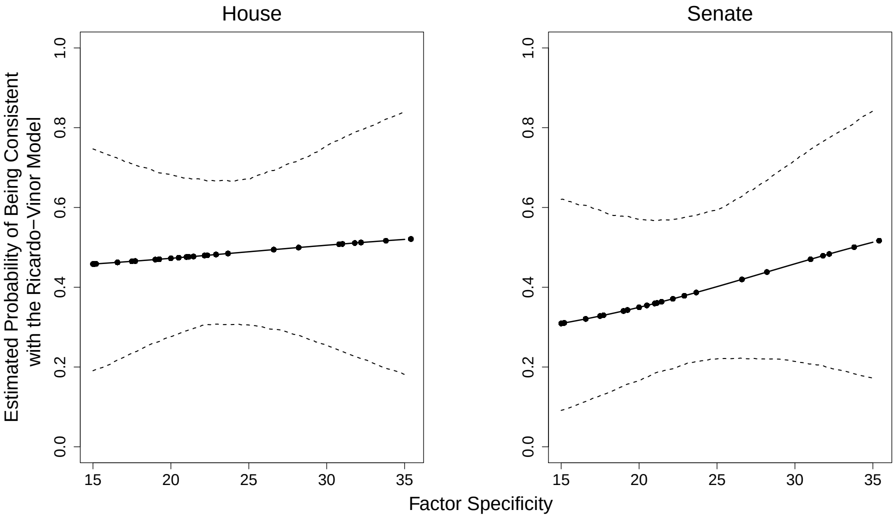
- Ricardo-Viner Model:

$$\text{logit}^{-1}(\gamma_0 + \gamma_1 \texttt{export}_{ij} + \beta_2 \texttt{import}_{ij})$$

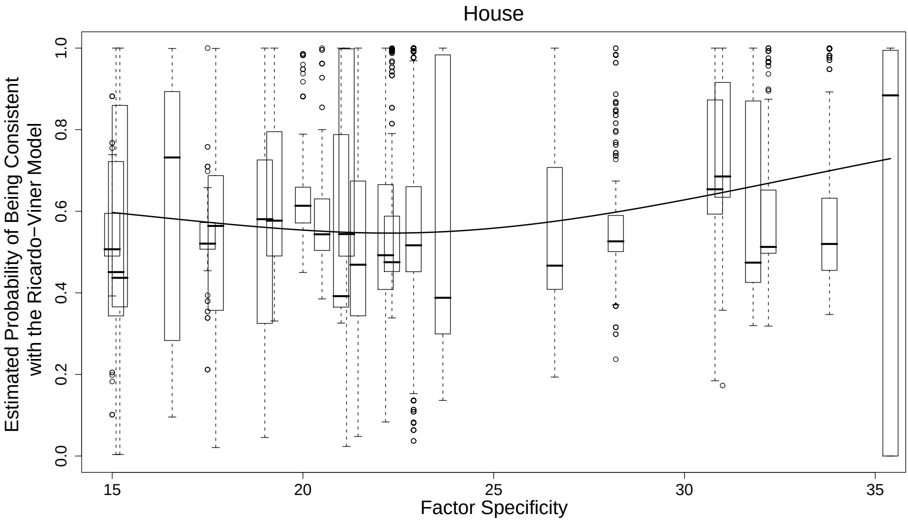
- Model for mixing probability:

$$\text{logit}^{-1}(\delta_0 + \delta_1 \texttt{factor}_j)$$

# Results with Grouped Observations



# Results without Grouping and Parametric Assumption



# Mixture Model vs. Garbage-can Model

| | | **Mixture Model** | | | | **"Garbage-can" Model** | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | House | | Senate | | House | | Senate | |
| Models | Variables | coef. | s.e. | coef. | s.e. | coef. | s.e. | coef. | s.e. |
| SS | profit | −1.60 | 0.53 | −5.69 | 1.19 | −0.42 | 0.33 | −2.14 | 0.73 |
| | manufacture | 17.60 | 1.54 | 19.79 | 2.59 | 5.69 | 0.63 | 4.73 | 1.32 |
| | farm | −1.33 | 0.29 | −1.27 | 0.43 | −0.11 | 0.14 | −0.03 | 0.25 |
| RV | import | 3.09 | 0.33 | 2.53 | 0.80 | 0.63 | 0.21 | 1.21 | 0.43 |
| | export | −0.85 | 0.16 | −2.80 | 0.77 | −0.85 | 0.08 | −1.48 | 0.20 |
| $\pi$ | factor | 0.01 | 0.06 | 0.05 | 0.07 | | | | |

- All estimates have expected signs and are statistically significant for the mixture model
- Garbage-can regression has smaller and sometimes statistically insignificant coefficients
- The original analysis contains some estimates with "wrong" signs

# Classification of House Trade Bills

| Stolper-Samuelson Model | Ricardo-Viner Model |
|---|---|
| Adams Compromise (1832) | Tariff Act (1824) |
| Clay Compromise (1833) | Tariff Act (1828) |
| Tariff Act (1842) | Gorman Tariff (1894) |
| Walker Act (1846) | Underwood Tariff (1913) |
| Tariff Act (1857) | RTAA (1934) |
| Morrill Act (1861) | RTA Extension (1937) |
| Tariff Act (1875) | RTA Extension (1945) |
| Morrison Bill (1984) | RTA Extension (1955) |
| Mills Bill (1988) | Trade Expansion Act (1962) |
| McKinley Tariff (1890) | Mills Bill (1970) |
| Dingley Tariff (1894) | Trade Reform Act (1974) |
| Payne-Aldrich Tariff (1909) | Fast-Track (1991) |
| Fordney-McCumber Tariff (1922) | NAFTA (1993) |
| Smoot-Hawley Tariff (1930) | GATT (1994) |
| Trade Remedies Reform (1984) | |

- Fitting the SS (RV) model to the SS and RV votes separately reveals an interesting pattern in terms of sign and statistical significance of estimated coefficients

# Concluding Remarks

- Mixture models offer an effective way to test competing theories
- Particularly useful in observational studies when causal inference is difficult but predictive inference is possible

- Many advantages over the standard model selection procedures:
  1. Test any number of competing theories
  2. Include nested and/or non-nested models
  3. Conduct frequentist or Bayesian inference
  4. Quantify the overall performance of each theory
  5. Test the conditions under which each theory applies
  6. Identify observations statistically significantly consistent with theory

- Some potential pitfalls:
  1. Demands more from the data
  2. Computationally intensive
  3. Lack of statistical power